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Ionospheric H-Atom Tomography: a Feasibility Study using GNSS Reflections.

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Goals of the GIOS-1 study

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GIOS: Global/GNSS Ionospheric Observation System

- To carry out a simple feasibility study to assess the potential of GNSS-R data for Space Weather applications (based on ionospheric electron content using GNSS signals of opportunity).
- To provide a proof-of-concept that GNSS-R ionospheric TEC data can have significant impact by providing TEC data over the oceans of vertical nature.
- *There is previous work on the use of GNSS-G and GNSS-ON data for tomography, but none so far with GNSS-R.*
- *In order to provide good tomographic solutions, data is need in all 3 spatial directions. Over the oceans, GNSS-G data is missing (only GNSS-ON data is forseen, of horizontal nature).*

The GNSS TEC Basic Equations

GPS observables consist essentially of the delays experienced by the dual frequency signals (f_1 =1.57542 GHz and f_2 =1.22760 GHz) transmitted from the GPS constellation (25 satellites) and received at GPS receivers around the world and in orbit. Let L_i be the measured total flight time in light-meters of a ray going from a given GPS satellite to a receiver at the frequency f_i

$$I = \int_{ray} dl\, \rho(\vec{x})$$

the integrated electron density along the ray (in electrons per square meter).

$$L_i = D - I\,\alpha / f_i^2 + \tilde{c}_{sat} + \tilde{c}_{rec}$$

More basic equations...

For the purposes of ionospheric analysis the D_I combination is formed:

$$\begin{aligned} D_I &= D_1 - D_2 \\ &\approx \alpha \left(\frac{1}{f_1^2} - \frac{1}{f_2^2} \right) \int_{s.l.} dl\, \rho(\vec{x}) + \tilde{c}^I_{sat} + \tilde{c}^I_{rec} + \text{noise}, \end{aligned}$$

where ρ is the ionospheric electron density and the new delay constants are given by the difference between the corresponding constants at each frequency. Then we can write

$$\sigma^2_{D_I} = \sigma^2_{D_1} + \sigma^2_{D_2}$$

Let us consider for now that the ranging error in each of the frequencies is similar. Then we can write,

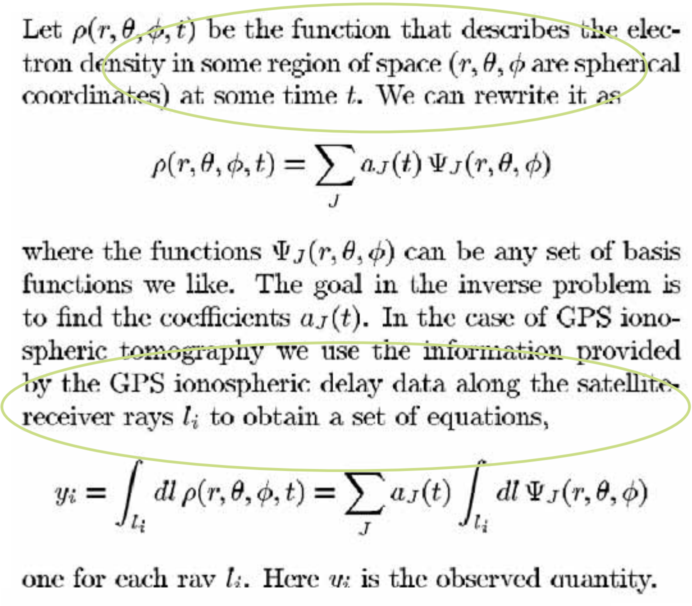
$$\sigma_{D_I} = \sqrt{2} \cdot \sigma_D$$

$$L = \gamma\, I + c_{sat} + c_{rec}$$

$\gamma = 1.05 \times 10^{-17} \;\; m^3$

$\alpha = 40.3\, m^3/s^2$

Early work...



Let $\rho(r,\theta,\phi,t)$ be the function that describes the electron density in some region of space (r,θ,ϕ are spherical coordinates) at some time t. We can rewrite it as

$$\rho(r,\theta,\phi,t) = \sum_J a_J(t)\,\Psi_J(r,\theta,\phi)$$

where the functions $\Psi_J(r,\theta,\phi)$ can be any set of basis functions we like. The goal in the inverse problem is to find the coefficients $a_J(t)$. In the case of GPS ionospheric tomography we use the information provided by the GPS ionospheric delay data along the satellite-receiver rays l_i to obtain a set of equations,

$$y_i = \int_{l_i} dl\,\rho(r,\theta,\phi,t) = \sum_J a_J(t) \int_{l_i} dl\,\Psi_J(r,\theta,\phi)$$

one for each ray l_i. Here y_i is the observed quantity.

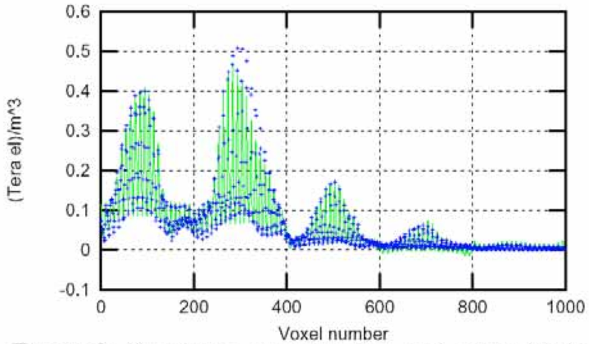
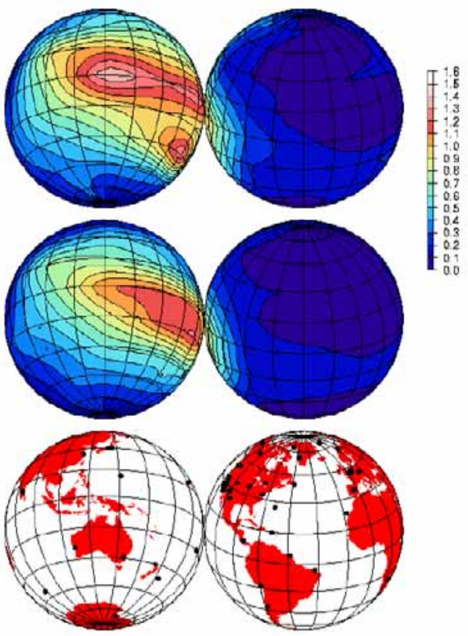


Figure 2: *Simulation reconstruction and original field (crosses). Sub-T/P bias (up to voxel 800) and rms are 0.26 and 0.70 TECU, respectively. Super-T/P bias and rms are 0.30 and 0.6 TECU.*

2 Layer model

Using GNSS-G



5 Layer model

Using GNSS-G/ON

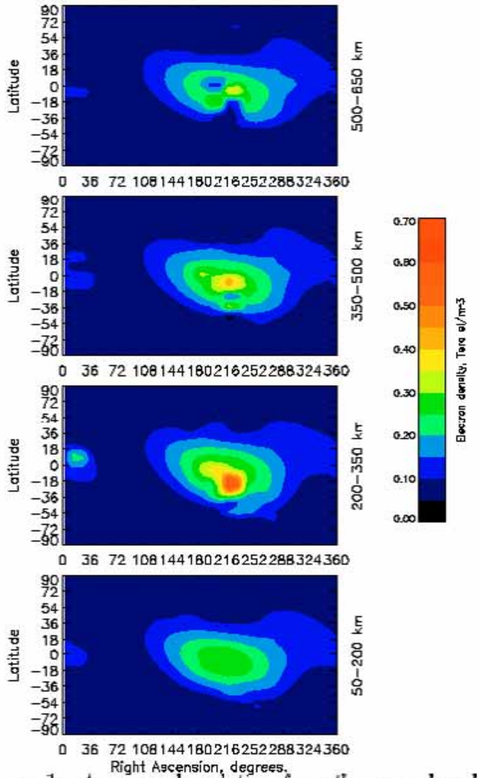
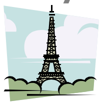


Figure 1: *An example solution from the ground and Low Earth Orbiter (GPS/MET) combined data, UT 21-24, Nov 2nd, 1996. In these coordinates, the Sun is at 217° Right Ascension, and -14° declination.*

GNSS-R Basics

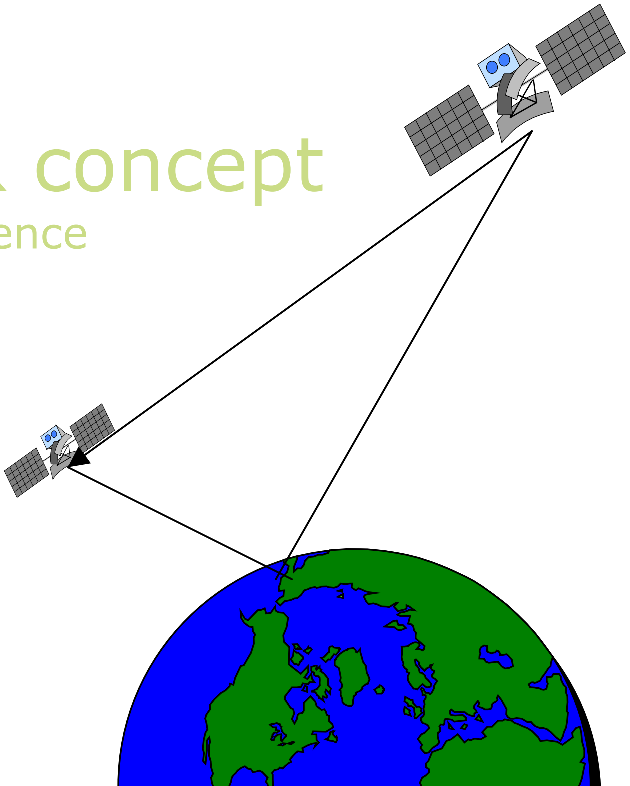
- GNSS-R= EO using GNSS signals of opportunity:
  - Altimetry: PARIS (Paris-n ESA projects)
  - Speculometry (roughness) (Oppscat-n ESA projects)
  - TEC (GIOS-1 ESA Study)



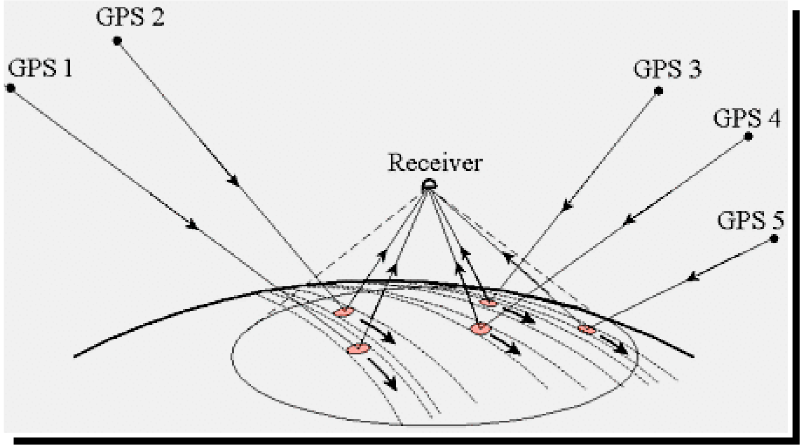
- Multi-static Passive Radar
- All-weather Multi-frequency system (L-band)
- Direct vs reflected signal differencing scheme = error cancellation
- Earth coverage, space and time resolution suitable for studying mesoscale properties of the ocean

Earth Observation: GNSS-R concept

Using GNSS Reflections for Earth Science

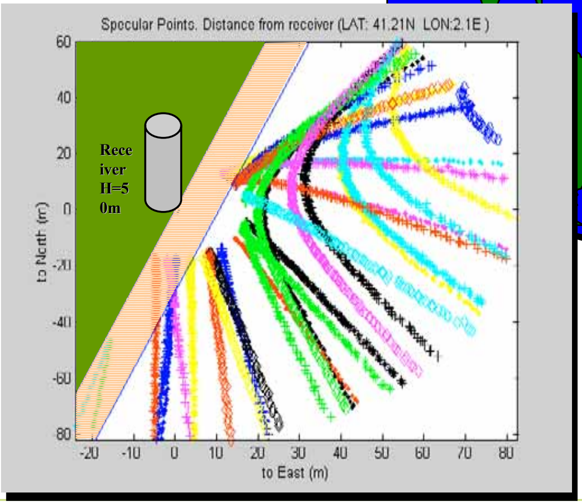


Altimetry and "speculometry" for global and <u>mesoscale</u> monitoring

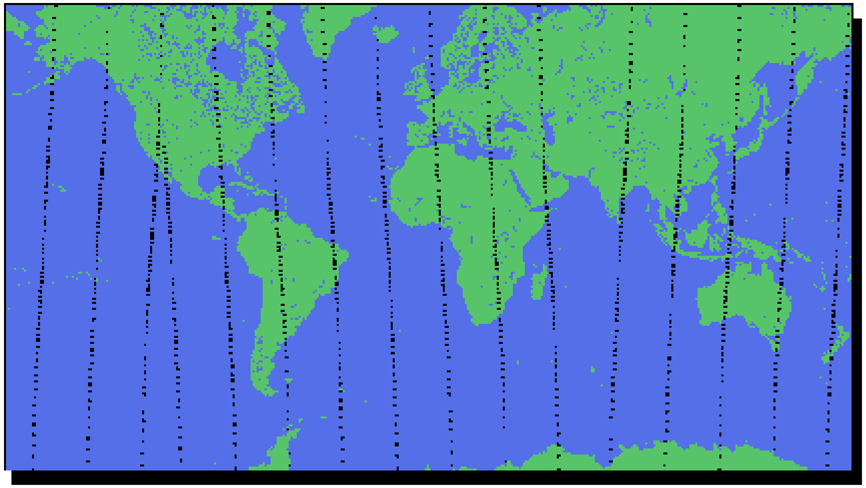


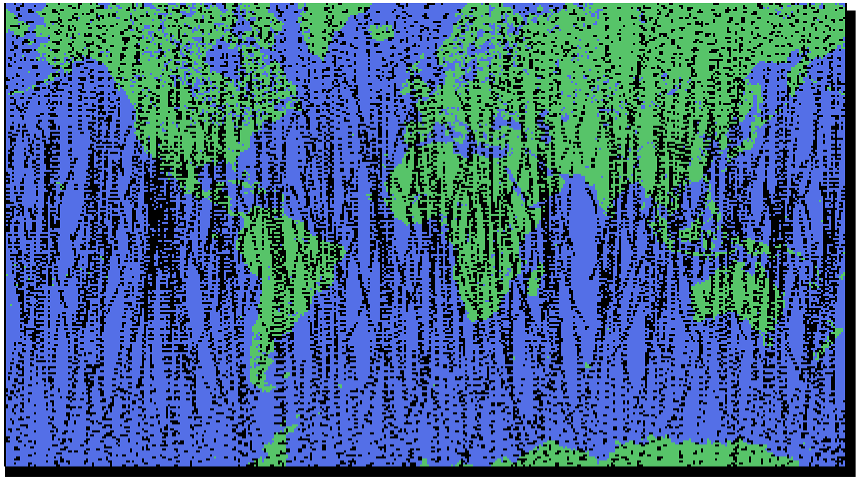
Passive, Multistatic, Stable

Reflection seascape from Mapfre Tower in Barcelona, using 24 h of GPS signals.



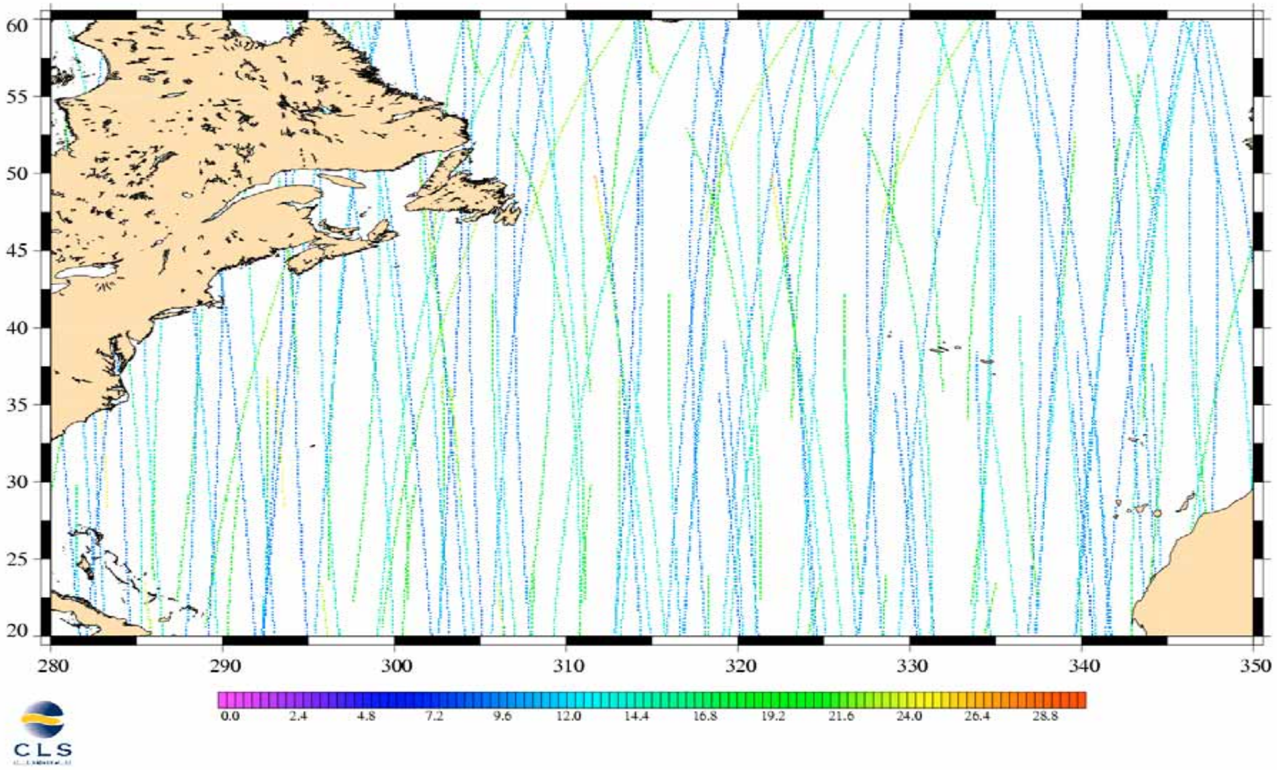
Multistatic advantages





CHAMP scenario, 12 h

(from the OPPSCAT 1 project)

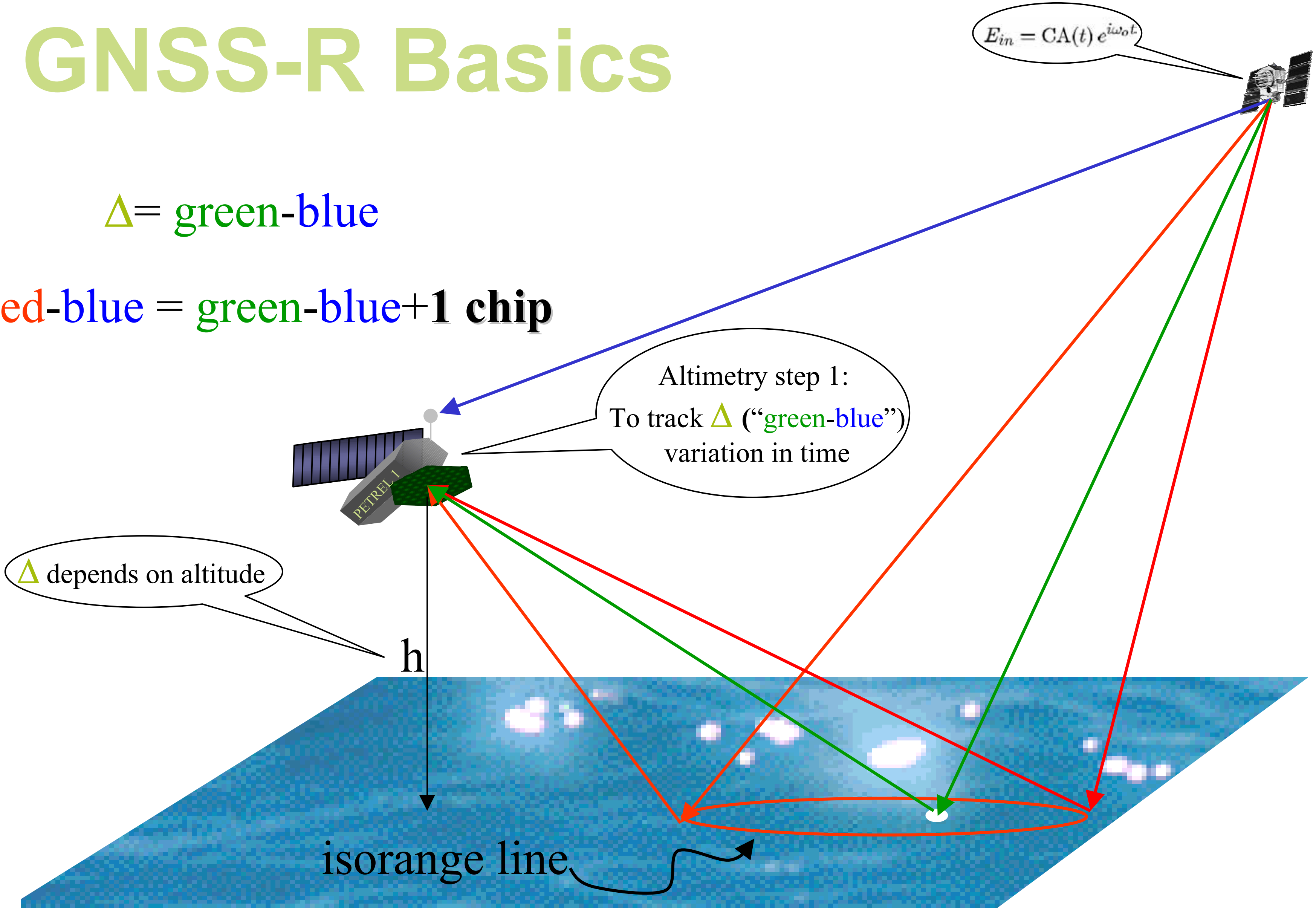


3-day sampling of the North Atlantic from one LEO GPS receiver (6 beams) and the September 15, 2001 GPS constellation with errors <u>for 3-second</u> averages (cm) [From Paris Beta Project]

GNSS-R Basics

Δ= green-blue

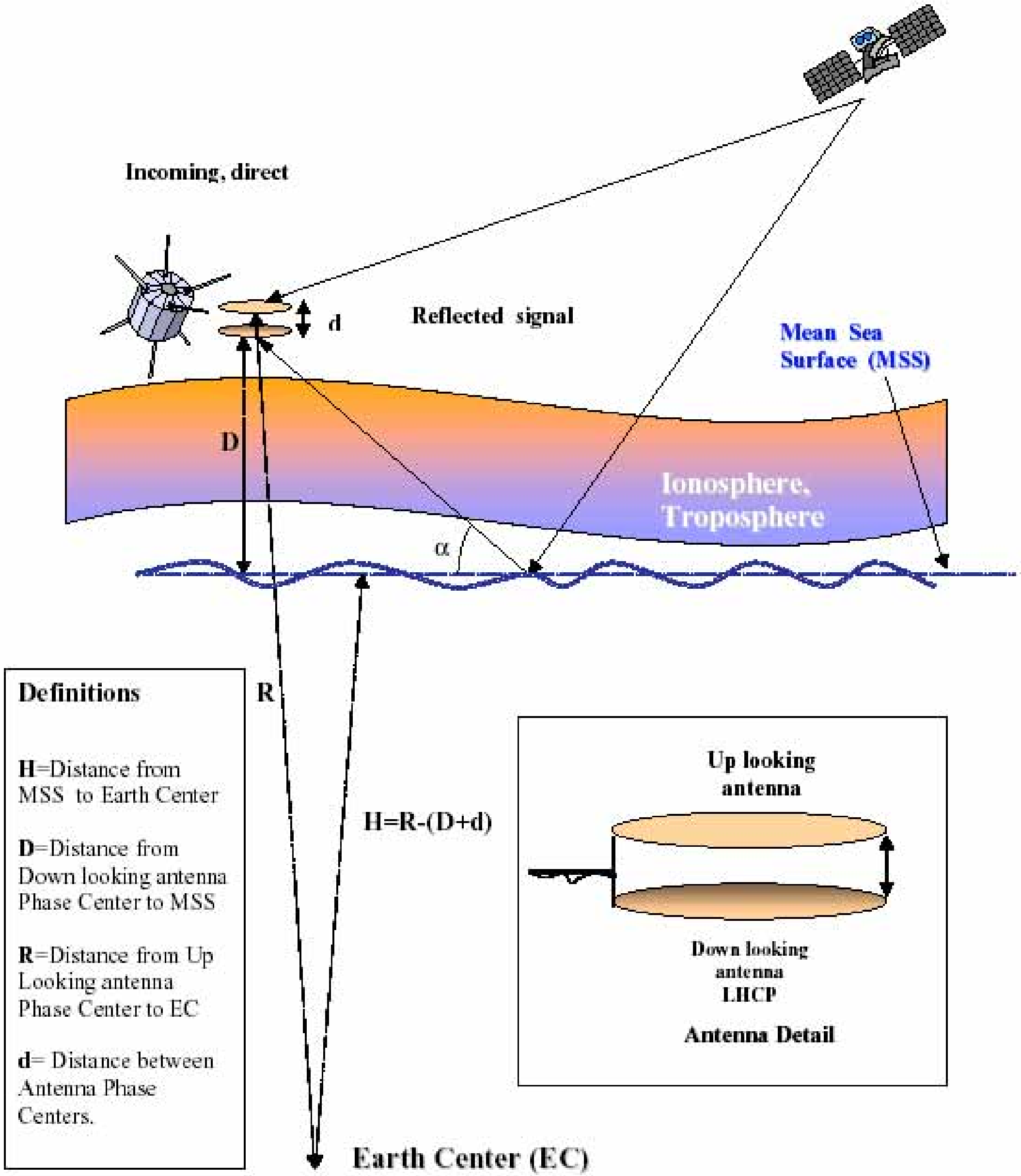
red-blue = green-blue+**1 chip**



$E_{in} = \mathrm{CA}(t)\, e^{i\omega_o t}$

Altimetry step 1:
To track Δ (“green-blue”)
variation in time

Δ depends on altitude



Incoming, direct

d

Reflected signal

Mean Sea Surface (MSS)

D

Ionosphere, Troposphere

α

R

$H=R-(D+d)$

Definitions

H=Distance from MSS to Earth Center

D=Distance from Down looking antenna Phase Center to MSS

R=Distance from Up Looking antenna Phase Center to EC

d= Distance between Antenna Phase Centers.

Up looking antenna

Down looking antenna LHCP

Antenna Detail

Earth Center (EC)

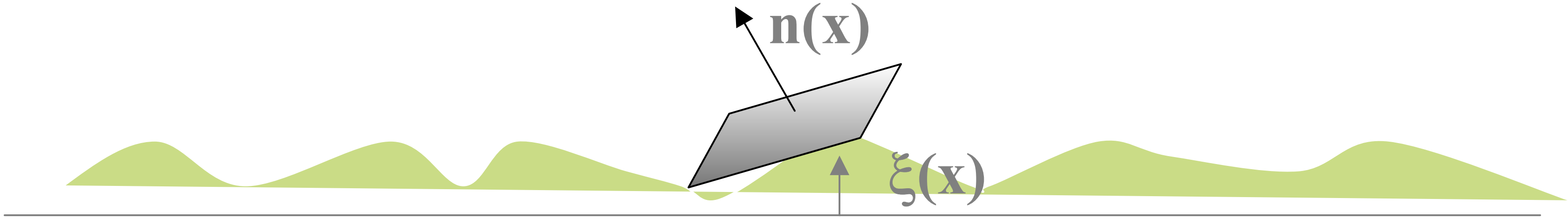
What GNSS-R can deliver

- *dmss*: Directional Mean Square Slopes,

  $<(\partial_i \xi)\,(\partial_j \xi)>$

- Altimetric Measurements: *h*
- biTEC, biTROP

- All at higher resolution than with any monostatic system.



Recent GPS-R experiments

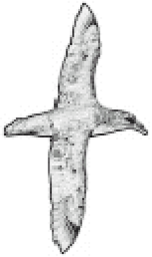
The Pond experiment, Jan 2000 was designed to test some key issues in the **PIP concept**. The PIP concept is based on the use of **dual-frequency carrier measurements** to exploit the correlations in the scattered signals at similar frequencies, an idea of Manuel Martin-Neira of ESA. PIP has been inspired from the Three Carrier Carrier Ambiguity Resolution concept, in the context of Galileo's high precision relative navigation research. The pond experiment was designed to use the fact that in a calm water environment, a normal GPS receiver can track the phase provided it is attached to an antenna of the right polarization.

BRIDGE 2, Jun 2001: A recent experiment to collect reflected L1 and L2 GPS-R data, together with more ancillary data such as RINEX, wind speed/direction/tide, SWH. (Data processing currently performed within the PARIS Alpha Project).

PARIS Alpha Flight, Sep 2001: GPS-R data collection from **airborne platform**. (Data processing currently performed within the OPPSCAT 2 Project).

Eddy Experiment, Sep 2002: GPS-R data collection from **airborne platform**. (Data processing currently performed within the Paris Gamma Project).

The Atmosphere-Ocean interface



- O-A coupling is a key element for climate, wave and ocean circulation models. The interface between the ocean and the atmosphere is the sea surface, which is characterized statistically, to the lowest order, by *h*, *swh* and *dmss*. Of these, *dmss* is one key missing element in GOOS, and is essential to understand and quantify the atmosphere-ocean flux of energy and momentum. Mesoscale measurements of *h* are also needed.

- Since inertial motion forcing is strongly intermittent in space and time, *x-t* collocated measurements of *h* and *dmss* are very relevant.

GIOS-1 Mission Design and Philosphy

- A near- polar LEO orbit is compatible with other applications requiring global coverage and moderately frequent revisits, such as mesoscale altimetry.
- Note however that the ionosphere is quasi-static as seen from an orbital inertial frame.
- A single LEO will only sample a ionospheric slice, but with good temporal sampling.
- We aim to show good results in such a slice, using only GPS data.

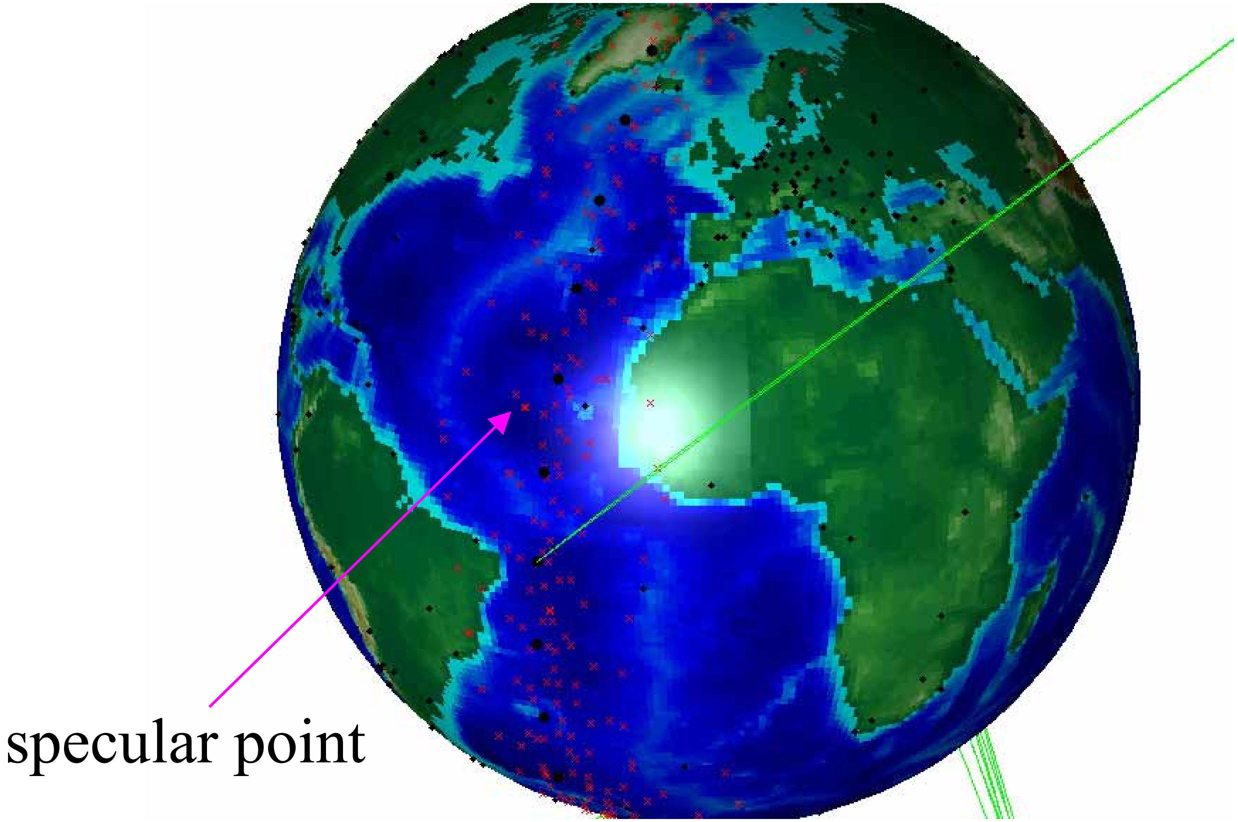
GIOS-1 Simulation of Data

- We simulated data as seen from a LEO orbiter able to collect GPS-R and GPS-Occultation/Navigation data.
- GPS orbital data taken from a standard Yuma file
- GPS-G data from > 300 IGS stations (vertical) also simulated.
- Gaussian noise added to slant TEC measurements: 10 TECU after 10 seconds averaging (although we did not see much impact from noise)
- In simulations, for simplicity we compressed input data to a cadence of 1-3 minutes (very conservative).

GIOS-1 Noise Model

- Current GNSS-R models indicate that with a 15 dB mission such as mPETREL, bistatic ranging errors in each channel would be of the order of 3 meter after 1 second.
- This translates into about 4 m ionospheric delay, or about 40 TECU. After 10 seconds, this is about 13 TECU.
- This is a worst case scenario, as current concepts involve >20 dB antennas.

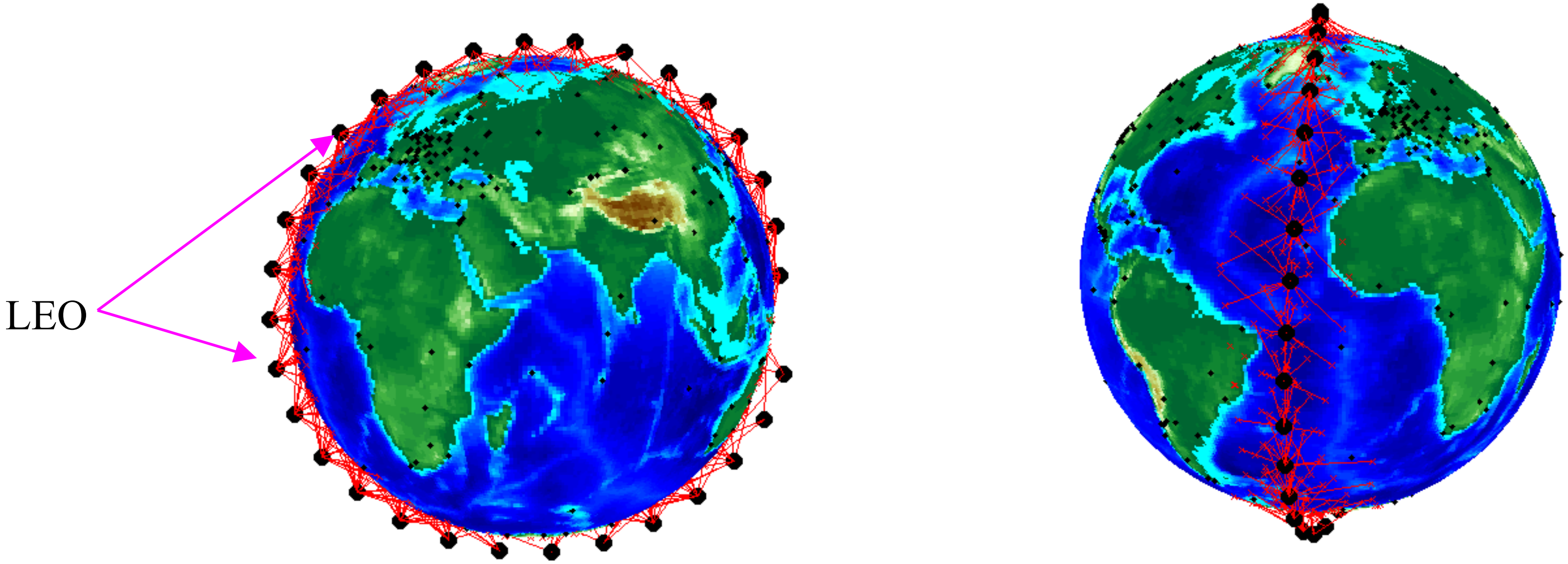
SIMULATING GPS TEC data 1



specular point

A grazing occultation ray link giving a particularly large slant TEC (about 600 TECU). Note that reflections can be observed on the surface, and that ground reflections have not been eliminated.

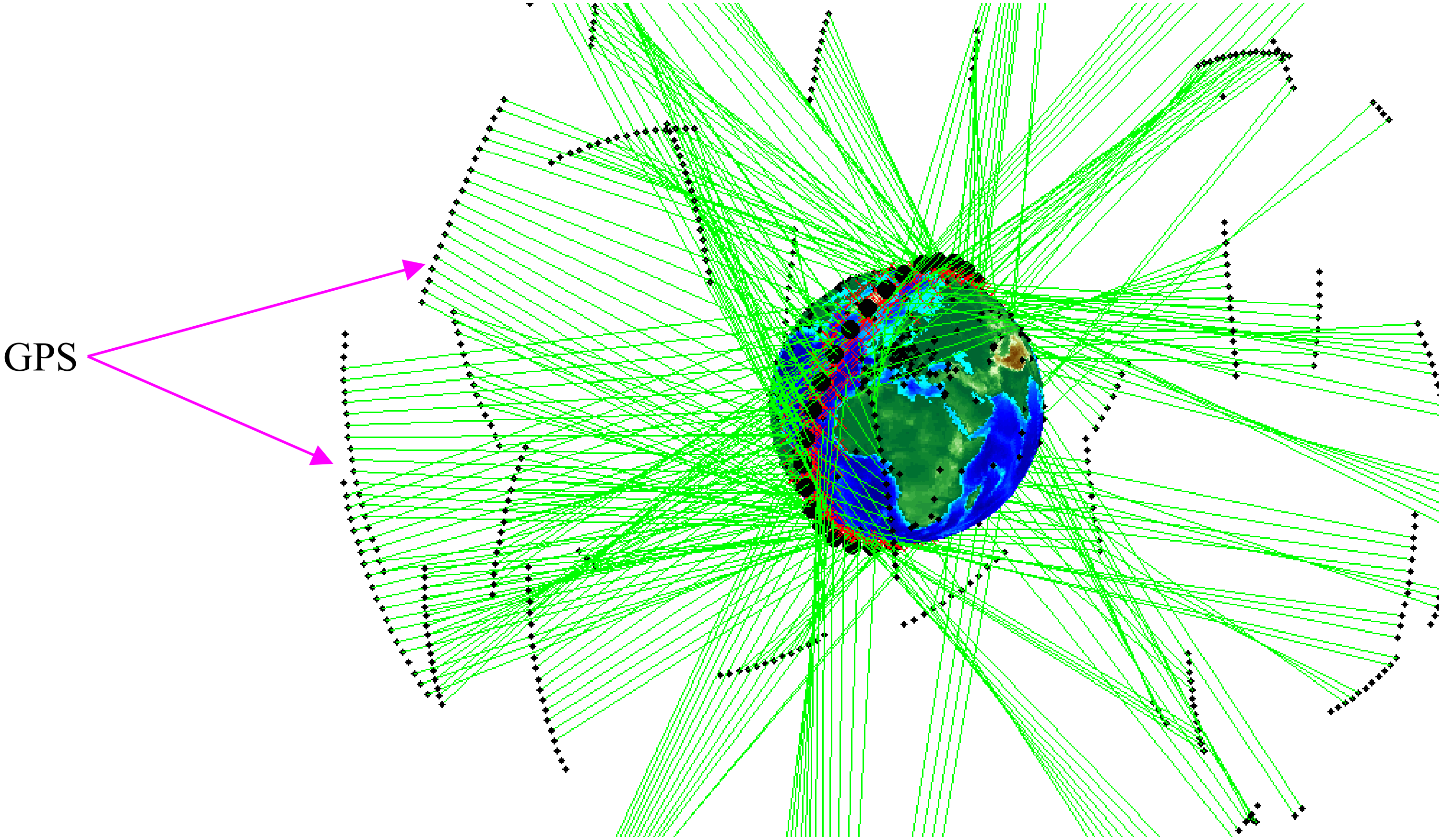
SIMULATING GPS TEC data 2



No cutoff angle has been used for reflections, only land masking has been applied.

LEO positions with a cadence of 3 minutes (orbit positions are shown as black dots). In addition, LEO to Earth Specular links (over the oceans) are shown in red. Ground stations are shown as black dots but no GPS-ground links are shown. Land reflections are identified and neglected (not linked). Ground stations are again shown as black dots but no GPS-ground links are shown.

SIMULATING GPS TEC data 3



GPS to LEO Navigation/Occultation ray links in green, as well as the LEO and GPS positions every 3 minutes. Ground stations are shown as black dots but no GPS-ground links are shown

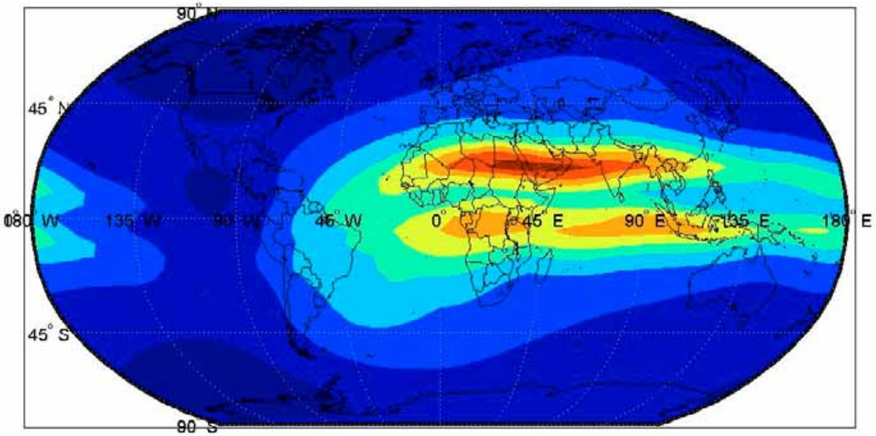
Simulation of Ionosphere

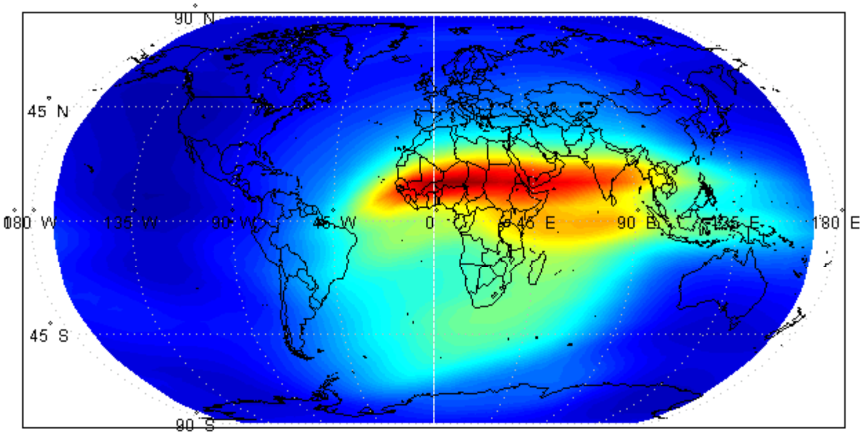
- Two climatic models of the ionosphere with good top side models have been considered, NeQuick and PIM.
- Finally, PIM was chosen (basically for ease on interfacing), but the results do not depend much on this choice.
- 3D ED fields with a resulution of 10 degrees and many (~ 50) layers up to GPS orbit where generated to compute simulated slant TECs.

IONOSPHERE ED MODELS: NeQuick AND PIM

NeQuick

PIM





Simulating TEC

- From orbit simulation, choose positions and integrate ED along ray.
- In bistatic case, this involves two ray segments for the reflected signal.
- Add Gaussian noise equivalent to 10 TECU after 10 seconds.

Tomography and the quantum H atom

- Some previous approaches have focused on voxel (3D pixels) representations- local support representations. This fact alone makes them awkward and inefficient when not the whole ionosphere is sampled-as will be the case here and rather generally.
- We have developed and worked with a new representation which we call the *H-representation*, related to the solutions to the Schrodinger equation for the Hydrogen atom.
- This representation is non-local. This means that if data is available at only specific regions of the ionosphere, all the coefficients in the representation can contribute to the fit. This allows for a good fit where there is data at the expense of sparsely sampled regions (which should not be trusted anyhow).
- The H-Representation also offers the advantage of easy integration of smoothing terms to account for data scarcity and mathematical elegance (simplicity).

$$\frac{-\hbar^2}{2\mu}\frac{1}{r^2\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right)+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right)+\frac{1}{\sin\theta}\frac{\partial^2\Psi}{\partial\phi^2}\right]$$

$$-U(r)\Psi(r,\theta,\phi)=E\,\Psi(r,\theta,\phi)$$

THE H-REPRESENTATION 1

In this approach:

$$ED_{n,l,m}(r,\theta,\phi) = R_{n,l}(r)Y_{lm}(\theta,\phi)$$

similar to the representation used in the Schrödinger solution to Hydrogen atom.

We have two parameters to determine: a_0 and the number of coefficients (determined by n).

THE H-REPRESENTATION 2

Here Y_{lm} are the spherical harmonics, and $R_{n,l}$ is the radial function:

$$R_{n,l}(r) = c_{n,l}\, e^{-\rho/2}\, \rho^{l+1} L^{2l+1}_{n-l-1}(\rho), \qquad \rho = \frac{2r}{na_0}$$

where $c_{n,l}$ are normalization coefficients. Similarly, L are the Laguerre polynomials

$$L_n^k(x) = \sum_{m=0}^{n} (-1)^m \frac{(n+k)!}{(n-m)!(k+m)!m!} x^m$$

THE H-REPRESENTATION 3

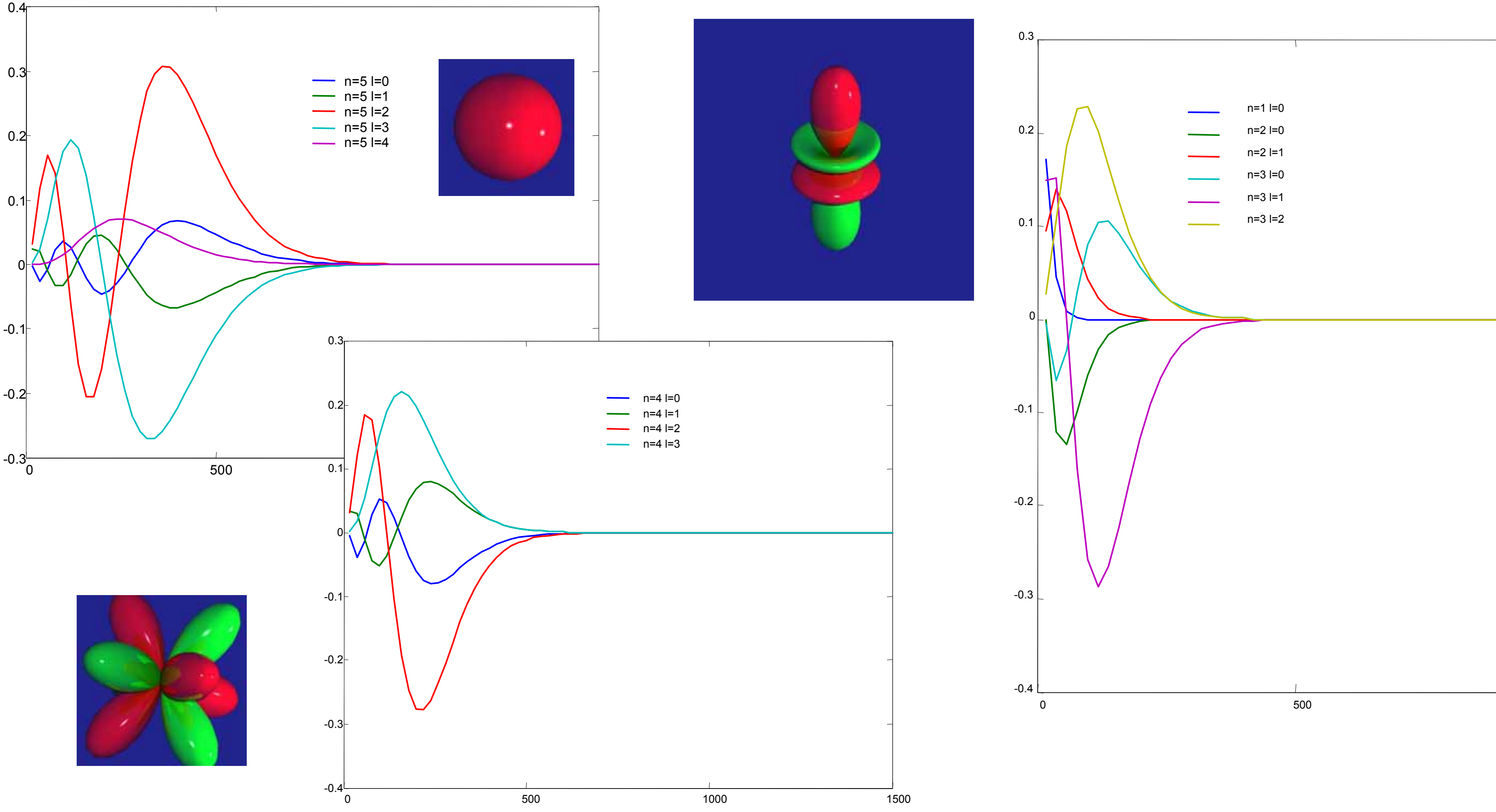
TEC= M·x, where x represents the unknowns in the expansion:

$$x = \begin{pmatrix} a_{100} \\ a_{200} \\ a_{210} \\ a_{211R} \\ a_{211I} \\ \vdots \end{pmatrix}$$

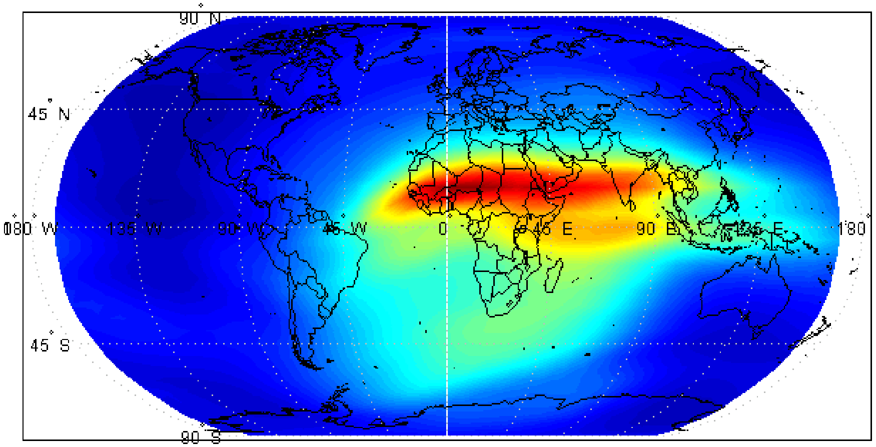
$$TEC = \gamma \int ED \cdot dl = \sum_n \sum_l \sum_m a_{nlm} \int f(r,\theta,\phi)\, dl$$

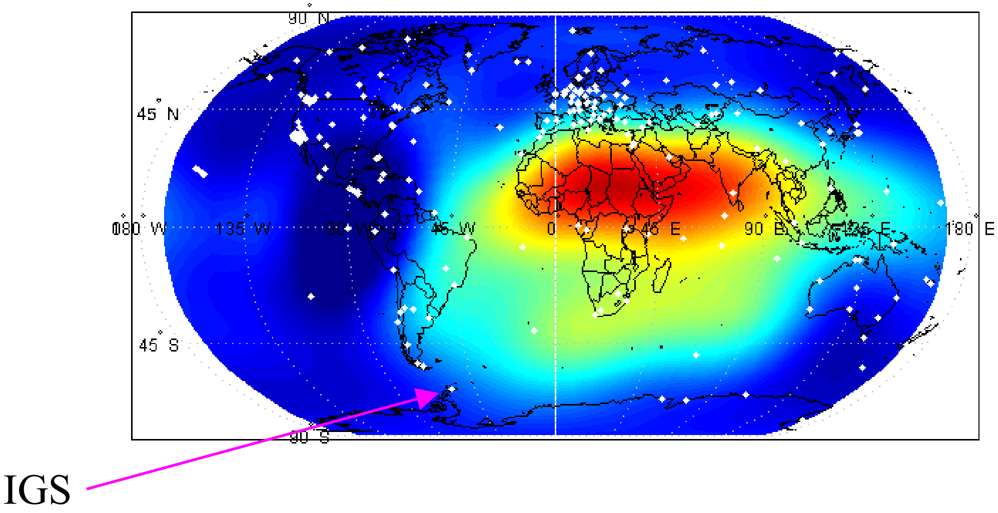
$$M = \begin{pmatrix} \int R_{10}(r)Y_{00}(\theta,\phi)dl_1 & \int R_{20}(r)Y_{00}(\theta,\phi)dl_1 & \int R_{21}(r)Y_{10}(\theta,\phi)dl_1 & \mathrm{Re}\left(\int R_{21}(r)Y_{11}(\theta,\phi)dl_1\right) & \mathrm{Im}\left(\int R_{22}(r)Y_{11}(\theta,\phi)dl_1\right) & \cdots \\ \int R_{10}(r)Y_{00}(\theta,\phi)dl_2 & \int R_{20}(r)Y_{00}(\theta,\phi)dl_2 & \int R_{21}(r)Y_{10}(\theta,\phi)dl_2 & \mathrm{Re}\left(\int R_{21}(r)Y_{11}(\theta,\phi)dl_2\right) & \mathrm{Im}\left(\int R_{22}(r)Y_{11}(\theta,\phi)dl_2\right) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \int R_{10}(r)Y_{00}(\theta,\phi)dl_n & \int R_{20}(r)Y_{00}(\theta,\phi)dl_n & \int R_{21}(r)Y_{10}(\theta,\phi)dl_n & \mathrm{Re}\left(\int R_{21}(r)Y_{11}(\theta,\phi)dl_n\right) & \mathrm{Im}\left(\int R_{22}(r)Y_{11}(\theta,\phi)dl_n\right) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

Radials, Spherical Harmonics



The value of ground data (n=8)



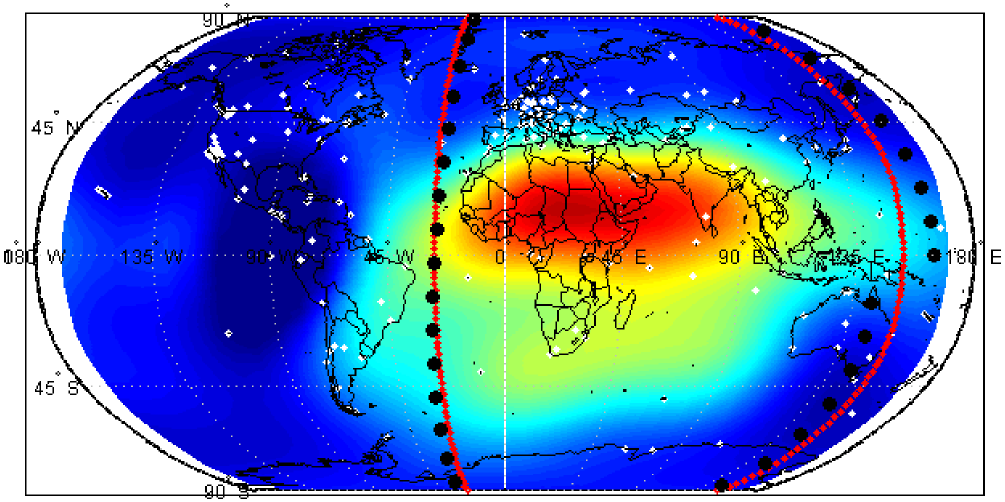


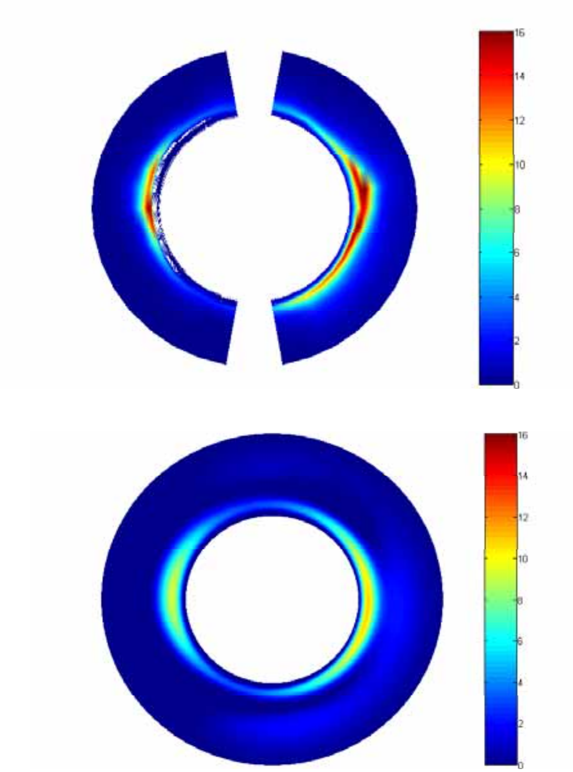
“Truth” from PIM

Recovered n=8 a_0=20 km

Right: Recovered TEC with n=8 (204 coefficients, a=20), using ground data (IGS stations appear as white dots) and all LEO data (over 10,000 measurements). Residual variance is of 6.3 (slant) TECU, mainly due to model “quantisation”. On the left we see the PIM original ionospheric TEC.

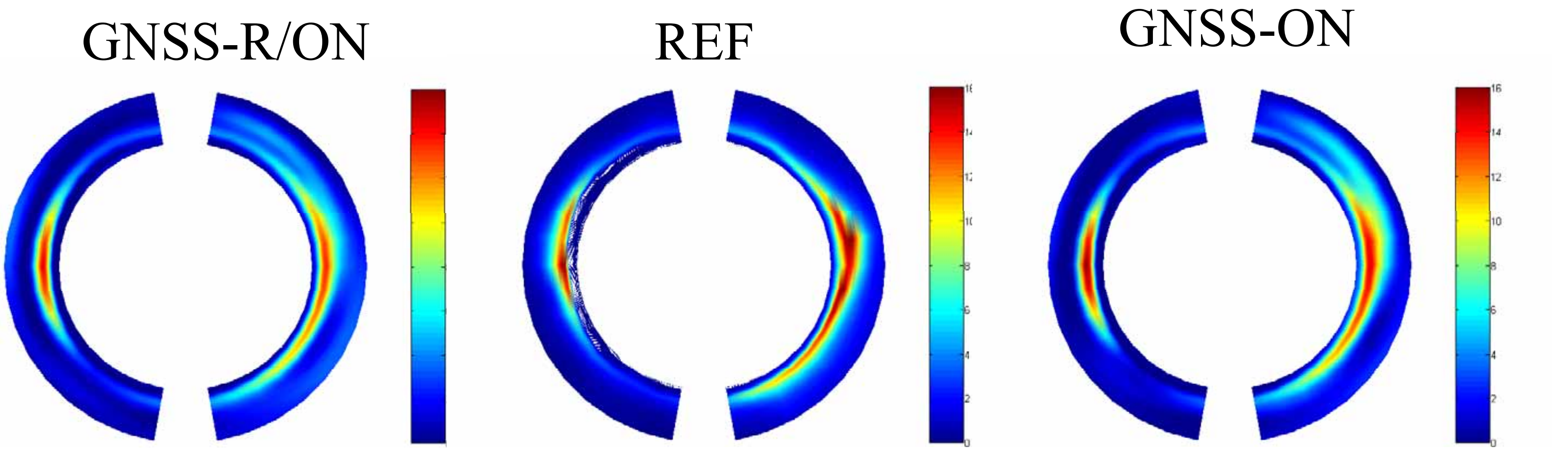
ED “slices”





Right: Orbital plane slice of reference (top) and recovered solution (bottom), using only GNSS-ON and GNSS-R data (no ground), and **n=5** (55 unknowns). Slice represents 90-2,000 km altitude. Note that the recovered solution is rather smooth, as it should, given the small number of variables used (yet, the fit residuals are already of the order of 15 TECU).

The value of GNSS-R data (n=8)

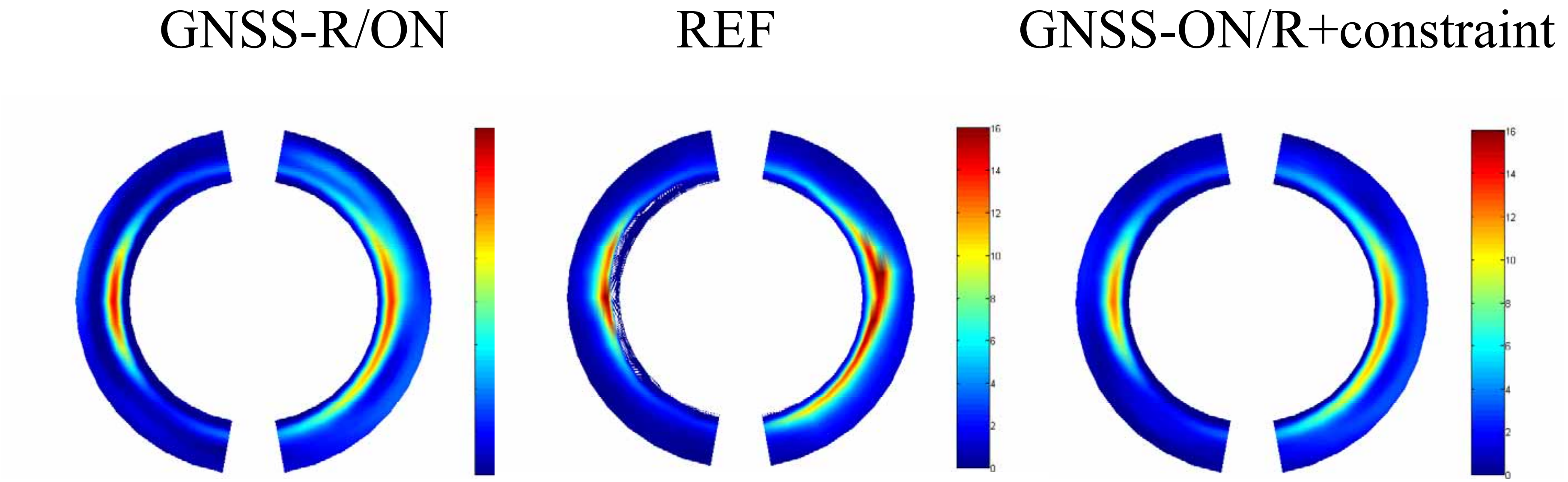

GNSS-R/ON

REF

GNSS-ON

Solution with an H-representation of order n=8 (204 unknowns, a=10 km) with GNSS-R data (**left**, 0.15 Tera el/m^3 mean error), reference truth (**middle**) and without GNSS-R data (0.16 Tera el/m^3 mean error). The GNSS-R data contribution (of vertical nature) is crucial to provide horizontal resolution: the GNSS-ON only solution is smoothed horizontally. No ground data has been used. Altitude spans from 0 to 1,000 km of altitude. Units are ED/1d11 electrons per cubic meter.

The value of constraints



GNSS-R/ON

REF

GNSS-ON/R+constraint

Solution with an H-representation of order **n=8** (204 unknowns, a=10 km) with GNSS-R data (**left**, 0.15 Tera el/m^3 mean error), reference truth (**middle**) and with a n-constraint (of λn^2 type) (0.14 Tera el/m^3 mean error). No ground data has been used. Altitude spans from 0 to 1,000 km of altitude. Units are ED/1d11 electrons per cubic meter.

TOMOGRAPHIC RESULTS

n	# coef	a_0 (km)	χ TEC (TECU)	χ ring ED (Terael/m^3)
5	**55**	**30**	**15.75**	**0.19/0.19**
6	**91**	**10**	**11.35**	**0.35/0.45**
6	**91**	**20**	**10.20**	**0.17/0.20**
8	**204**	**10**	**5.33**	**0.15**
8	**204**	**20**	**6.34**	**0.27**

We show the H-order and number of coefficients, the a_0 parameter, the slant TEC fit, and the ring fit, with/without GNSS-R data (for the first three only).

Conclusions

- The H-representation provides an efficient way to represent the solution space.
- With as little as n=5 (55 coefficients) we obtained a fit of 7 TECU under the LEO track, using only LEO data, while with the addition of IGS ground data gave a fit of about 13 TECU.
- The addition of ground data from a few stations provided a better global fit, as expected.
- GNSS-R data improves significantly the ED results on the Orbital Ring.

- *The addition of GNSS-R data can cover a crucial gap over the oceans, where "ground" (vertical) data is not available.*
  - *An interesting tomographic approach has been developed and implemented.*

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Abstract

- Within the context of the GIOS-1 ESA project, we have analyzed the feasibility of ionospheric monitoring using GNSS technology. We have focused on the use of LEO GNSS data, exploiting GNSS Reflections, Navigation and Occultation TEC measurements. The basic question we have addressed in this initial exploratory study is: can we provide a proof-of-concept that GNSS-R data will have a realistic impact on GNSS ionospheric modelling?
- In order to attack this question, we have simulated GNSS ionospheric TEC data as it would be measured from a polar LEO (exploiting Navigation, Occultation and Reflection TEC data) and IGS ground stations, through the use of a climatic ionospheric model (we have explored both NeQuick and PIM). We have developed a new tomographic approach inspired on the physics of the hydrogen atom, which has been employed to retrieve the Electronic Density field from the simulated TEC data for impact assessment.
- Tomographic inversion results using simulated data demonstrate the significant impact of GNSS-R and GNSS-NO data: 3D ionospheric Electron Density fields are retrieved over the oceans quite accurately, even as, in the spirit of this initial study, the simulation and inversion approaches avoided intensive computation. We conclude that GNSS-R data can contribute significantly to the GIOS (Global/GNSS Ionospheric Observation System). We provide ideas for further work, starting from a clear identification of users and their requirements, the development of more sophisticated simulations, further development of algorithms, analysis of LEO data, etc.