4D Imaging of the Ionosphere using GPS data and an Application to Ionospheric Calibration of Radar Altimeters

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Abstract

We show how it is possible to combine GPS/MET occultation data with ground data collected from 100+ IGS stations to perform stochastic tomography of the ionospheric electron content with a 3D global grid of voxels extending up to 2000 km above the mean surface of the Earth, and thus produce temporal series of 3D images of the ionospheric electron content. A correlation functional approach that enforces smoothness of the images is used, and a Kalman filter is used to assimilate the data and propagate the solutions in the time direction. Even with Anti-Spoofing on, we have checked that the combination of ground and occultation data and the use of smoothing techniques is robust enough for vertical resolution in a multi-layer model analysis. This has been demonstrated with simulations and the method has been tested by contrasting the GPS estimates with independent sources of data. We discuss as well some of the work we have performed in the area of GPS data ingestion in the Parametrized Ionospheric Model (PIM). As an application, we compare TEC measurements from the NASA Radar Altimeter and DORIS instrument on board TOPEX/POSEIDON with GPS TEC estimates, and evaluate different GPS data analysis strategies. We verify that global tomographic GPS analysis using a voxel grid is well suited for ionospheric calibration of altimeters. We show that a 1-day fit of 20-second-averaged NRA ionospheric correction data versus GPS tomographic TEC data has a bias of 3.4 TECU and a root mean square deviation of 3.2 TECU.

1 Introduction

The goals of this work are as follows:

- To discuss how we obtain 4D fields of electronic density from GPS ionospheric delays, combining ground and GPS/MET delay data in our tomographic analysis software.
- To demonstrate the ingestion of GPS (IGS and GPS/MET) data into a parameterized ionospheric model, selecting the parameters that minimize a suitable cost functional. Thus, GPS data is used to select dynamically the best model parameters, in a 3DVAR fashion.
- To compare TEC measurements from the NASA Radar Altimeter and DORIS instruments on board TOPEX/POSEIDON (T/P) with our GPS TEC estimates, and use the latter for calibration of the Radar Altimeter to the 1 TECU level.

GPS signals suffer a delay due to the electronic content of the atmosphere which can be separated from other effects and measured precisely. We will show how it is possible to combine GPS/MET occultation data with ground data collected from 100+ IGS stations around the world to perform stochastic tomography of the ionospheric electron content with a 3D global grid of voxels extending up to 2000 km above the mean surface of the Earth, and thus produce temporal series of 3D images of the ionospheric electron content. We discuss as well

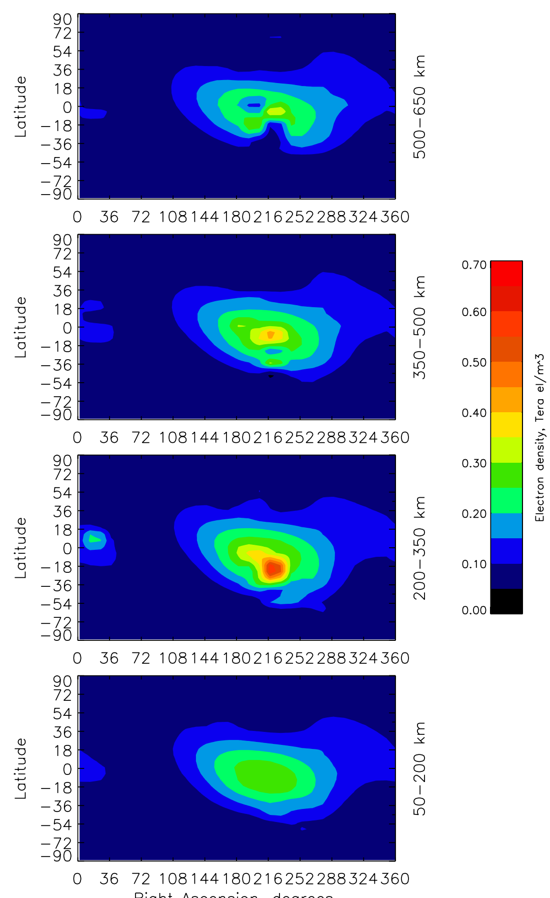

Figure 1: *An example solution from the ground and Low Earth Orbiter (GPS/MET) combined data, UT 21-24, Nov 2nd, 1996. In these coordinates, the Sun is at 217º Right Ascension, and -14º declination.*

some of the work we have carried out in the area of GPS data ingestion in the Parametrized Ionospheric Model (PIM) [6]. Ingestion of GPS data has been performed to select dynamically the best PIM parameters, in a 3DVAR fashion.

As an application, we compare TEC measurements from the NASA Radar Altimeter (NRA) and DORIS instrument on board TOPEX/POSEIDON with GPS TEC estimates, and evaluate different GPS data analysis strategies. We verify that global tomographic GPS analysis using a voxel grid is well suited for ionospheric calibration of altimeters. We show that a 1-day fit of 20-second-averaged NRA ionospheric correction data versus GPS tomographic TEC data has a bias of 3.4 TECU and a root mean square deviation of 3.2 TECU. Tomographic inversion using simulated data from the Parametrized Ionospheric Model (PIM) highlights the strong correlation between GPS bias constants, electronic densities at the highest layer, and unmodeled protonospheric TEC. This suggests that GPS TEC estimates at the TOPEX/POSEIDON altitude are more accurate if the bias constants are estimated and if a layer above TOPEX/POSEIDON is added to the grid.

2 Background

Let $\rho(r,\theta,\phi,t)$ be the function that describes the electron density in some region of space (r,θ,ϕ are spherical coordinates) at some time t. We can rewrite it as

$$\rho(r,\theta,\phi,t)=\sum_J a_J(t)\,\Psi_J(r,\theta,\phi)$$

where the functions $\Psi_J(r,\theta,\phi)$ can be any set of basis functions we like. The goal in the inverse problem is to find the coefficients $a_J(t)$. In the case of GPS ionospheric tomography we use the information provided by the GPS ionospheric delay data along the satellite-receiver rays l_i to obtain a set of equations,

$$y_i=\int_{l_i} dl\,\rho(r,\theta,\phi,t)=\sum_J a_J(t)\int_{l_i} dl\,\Psi_J(r,\theta,\phi)$$

one for each ray l_i. Here y_i is the observed quantity.

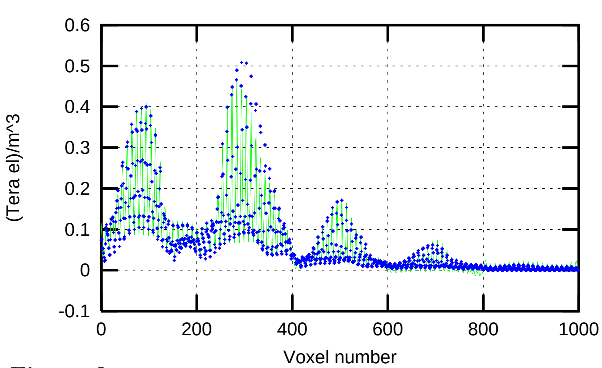

Figure 2: *Simulation reconstruction and original field (crosses). Sub-T/P bias (up to voxel 800) and rms are 0.26 and 0.70 TECU, respectively. Super-T/P bias and rms are 0.30 and 0.6 TECU.*

This is a set of linear equations of the form $Ax=y$, where the components of the vector x are the unknown coefficients $a_J(t)$. Since this system of equations may not have a solution we seek to minimize the functional $\chi^2(x)$, where (assuming uncorrelated observations of equal variance) $\chi^2(x)=(y-Ax)^T\cdot(y-Ax)$. In practice we find that although the number of equations is much greater than the number of unknowns, the unknowns (i.e., the array x) are not completely fixed by the data. We use a correlation functional approach that enforces smoothness of the images, and a Kalman filter to assimilate the data and propagate the solutions in the time direction.

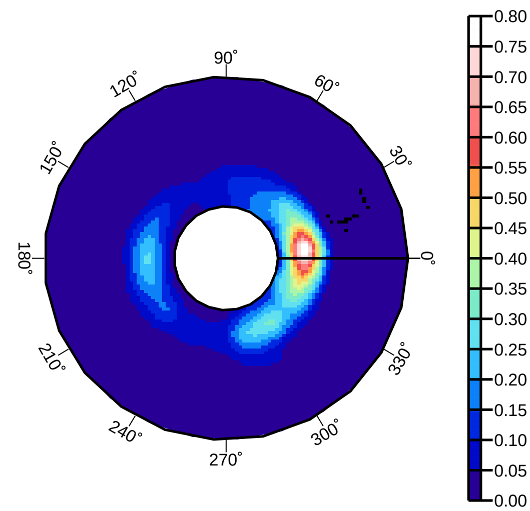

Figure 3: *GPS/MET orbit plane vertical slice through the ionosphere, with 6 layers (with boundaries at 6400, 6520, 6640, 6760, 6880, 7000 and 7700 km above the center of the Earth). Here, the 0^o point corresponds to the ascending node of GPS/MET. Longitude is then over the orbit. Units are Tera el/m^3.*

Climatological models of the ionosphere have existed for a while now, but it is only recently that they have been used to complement other sources of data, such as GPS, in the inversion process. The parameters controlling the model are normally input directly, however, and are not estimated themselves. *One could reason, however, that if the models were good enough they could be used to infer these parameters given other sources of data, such as GPS ionospheric delay data.* The resulting "best-fit" parameters should be closely related to the ones one can obtain by independent means. A climatological model such as PIM, maps the value of a set of parameters, λ_i, to the space $\{x\}$. Just as is done in variational weather modeling, we can picture minimizing the cost functional

$$J(\lambda_i)=\sum_j\left(O_j^{exp}-O[x(\lambda_i)]_j\right)^2,$$

where O_j^{exp} are the observables and $O[x(\lambda_i)]_j$ the modeled observables, in our case the slant delays produced by the ionospheric electrons.

Regardless of the technique used, Kalman filtering provides a natural way to enforce smoothness under time evolution, and is especially useful in the case of ionospheric stochastic tomography, when the "holes" in the information that we have at a given time (because of the particular spatial distribution of the GPS constellation and the receptor grid) may be "plugged" by the data from previous and future measurements.

In the context of PIM-fitting, we complement this step by using the previous solution in the iteration process to fit a PIM model to the data. In other words, if x_n and C_n are the solution and the covariance matrix at epoch n, we first determine a minimum squares PIM fit. Let A be the observation matrix (which we know how to compute, given a grid). Then we minimize the cost functional $J(\lambda) = \left(y - A\cdot x^{PIM}(\lambda)\right)^2$, and this will determine the PIM parameters λ^i, and the resulting image, $x_n^{PIM}(\lambda)$ and covariance matrix for the voxel image, C_n^{PIM}.

3 Using GPS data: Voxel and PIM tomography

GPS observables consist essentially of the delays experienced by the dual frequency signals (f_1 =1.57542 GHz and f_2 =1.22760 GHz) transmitted from the GPS constellation (25 satellites) and received at GPS receivers around the world and in orbit. Let L_i be the measured total flight time in light-meters of a ray going from a given GPS satellite to a receiver at the frequency f_i (including instrumental biases), and $I = \int_{ray} dl\, \rho(\vec{x})$ be the integrated electron density along the ray (in electrons per square meter).

Then L_i is modeled by $L_i = D - I\,\alpha/f_i^2 + \tilde{c}_{sat} + \tilde{c}_{rec}$, where $\alpha = 40.3\,m^3/s^2$, D is the length of the ray, and $\tilde{c}_{sat}$ and $\tilde{c}_{rec}$ are the instrumental biases. In the present case we are interested in the frequency dependent part of the delay: $L = L_1 - L_2$ (in meters). This is the derived observable and is modeled by ($\gamma = 1.05 \times 10^{-17}$ m^3) $L = \gamma\, I + c_{sat} + c_{rec}$, independent of D (see [2] for more details).

We have collected GPS data from GPS/MET and a subset of the International GPS Service (IGS) Network, for the day of February 23rd of 1997. This particular day has been chosen because of A/S is known to have been off. Geomagnetic and solar activity indices (as distributed by the US National Geophysical Data Center) for that day indicated a mean K_p index of 2.3, and $F_{10.7}$ = 73. The raw data has been pre-processed in order to obtain the observables using the procedures described in [2]. To describe the ionosphere we use five geocentric spherical layers beginning at 50 km above the mean surface (6350 km) of the Earth and extending 1400 km. Each layer consists then of two hundred voxels of dimensions 18^o in latitude, times 18^o in longitude, times 150 km of height for the first 4 layers. The unknowns here consist of the electron densities at each of these voxels, plus the unknowns corresponding to the transmitter and receiver constant delays. Figures 1 and 2 and 3 illustrate some of the results, while Figure 4 shows tomographic and PIM-fit residual histograms.

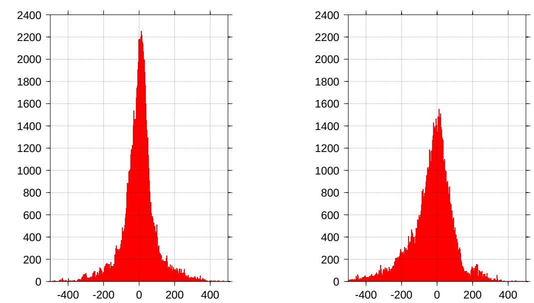

Figure 4: *Left: Tomographic residuals histogram. Standard deviation is 30 cm. Right: PIM-fit residuals (at $F_{10.7} = 52$ and $K_p = 0$). Standard deviation is 40 cm.*

4 Calibration of T/P

As mentioned at the begining, the electron content in the ionosphere produces delays in the phase and group propagation of radio waves. Thus, the operation of satellite radar altimeters is affected by the electronic distribution in the ionosphere. For this reason, some satellite altimeters such as the NRA aboard TOPEX/POSEIDON (T/P) operate at two frequencies (13.6 GHz and 5.3 GHz) and make use of the dispersive nature of ionospheric refractivity to correct for this effect. Unfortunately, because of its electronic nature, this correction needs to be calibrated. For the purpose of calibration, tomographical estimation of TEC should in principle be superior to estimating TEC using thin-shell mapping function techniques, since it allows greater freedom in the vertical distribution estimation. To test these ideas, IGS and GPS/MET low and medium rate data (in RINEX format) were processed for February 21st 1997 (A/S off) using our XT-GIST (Global Ionospheric Stochastic Tomographer) software package. We broke the flow of satellite delay data into three-hour blocks, and smoothness under time evolution was enforced using a Kalman filter. We performed the tomographic inversion in a $6 \times 10 \times 20$-voxel model (totaling 1200 unknowns excluding bias constants), with a resolution of 18^o in latitude and 18^o in longitude, and consisting of four 150 km-thick layers (extending from 6400 to 7000 km), plus a 700 km layer extending to the T/P orbit height (1336 km above equator, or about 7700 km from the center of the Earth), and then another layer of 700 km as a protonospheric buffer. AVISO/CNES NRA ionospheric correction and DORIS data were compared to our tomographic model estimates (in Figure 5, the NRA data has been smoothed using a 20 second window to eliminate sea-roughness induced noise). Thus, we are comparing the at-T/P-height TEC predictions from Tomographic

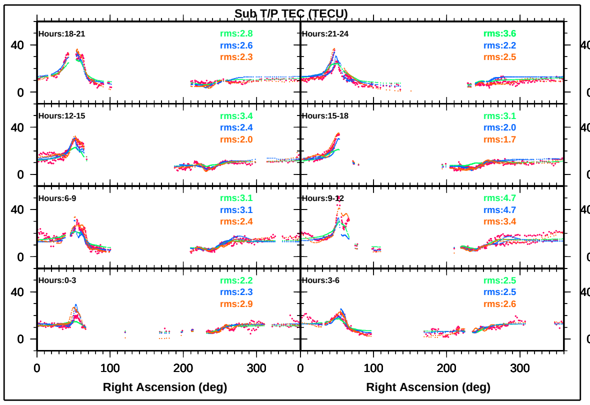

Figure 5: *TEC estimates from TGPS/3h, NRA, DORIS, and JPL, in eight 3-hour batches. The biases with respect to NRA have already been corrected.*

GPS (TGPS) and DORIS/Bent to the NRA retrieved TEC. As can be seen in Figure 5 and in the table, average TGPS TEC fits are only inferior to DORIS/Bent, while thin-shell models for GPS data analysis (such as CODE) yield a substantially worse fit. This improvement is expected, since DORIS measures ionospheric delays right where the altimeter is operating. DORIS and CODE estimates both use thin-shell models to extract TEC estimates, and such models are not very accurate for bias estimation.

For the purposes of calibration, it is best to use TGPS/24h: the bias obtained should be the same whether we use a mean or a high temporal rate solution TGPS solution, and the 24h solution is independent of the random walk drift rate or the *a priori* guess chosen in the Kalman filter. Restricting the reconstruction and comparison to the intersection of the T/P and GPS/MET orbits (see Figure 3) improves the TGPS/24h fit a little.

	Bias	**rms**
NRA-DORIS/Bent	4.4	3.0
NRA-Kalman	2.7	4.3
NRA-TGPS/24h	2.3	4.6
NRA-RTGPS/24h	3.0	4.6
NRA-CODE/24h	4.5	5.0

Table 1: *bias and rms of comparisons.*

5 Summary, Conclusions

In previous work we showed that ground and occultation GPS delay data can be combined successfully to perform ionospheric tomography with a substantial level of vertical resolution. Even with A/S interfering with phase alignment, and a less than optimal quantity of occultation data, our results provide evidence for both the need and substantial impact of occultation data in the reconstruction process.

We have seen here that such tomographic TEC estimates can be used for altimeter ionospheric calibration at the 1 TECU level using NRA and GPS data from one day. Moreover, we have provided evidence to show that bias constant estimation and an extra layer above the T/P orbit, despite being a computational nuisance, can improve the TEC estimates by absorbing the above-T/P protonospheric contribution to the GPS delays, and eliminating a potential source of bias in TGPS TEC retrievals. Our results thus suggest that TGPS can be used for absolute ionospheric calibration of radar altimeters.

We have also presented our efforts to use climatological models in tomographic analysis of GPS data. Climatological models such as PIM are essentially the result of performing Empirical Orthogonal Function analysis using observational or theoretically generated data, and in a way this is exactly what one would like to do in tomography: the basis functions used to span the space of possible solutions should be adapted to the field one is trying to map. Future efforts should be directed towards the development of more refined parameterized models.

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