3D GRADIENT EFFECTS ON TRANSIONOSPHERIC PATHS (SUCH AS FROM GPS) DETERMINED BY RAY-TRACING IN A 3D IRI IONOSPHERE

Karthigesu Nagarajoo
eenkn@leeds.ac.uk

Hal J. Strangeways
h.j.strangeways@leeds.ac.uk

School of Electronic and Electrical Engineering. University of Leeds. Leeds LS2 9JT. UK

Introduction

Numerical ray-tracing techniques are required to determine the effects of ionospheric horizontal gradients and the geomagnetic field on ionospherically reflected or transionospheric paths.

For very accurate ray-tracing in an anisotropic ionosphere with realistic 3D electron density gradients, the ionosphere model should be continuous function of the spatial co-ordinates and thus also the electron density. The spatial derivatives of the refractive index should also be continuous for first, second and higher order.

To perform precise ray-tracing in a 3D ionosphere model such as IRI, the vertical profile must first be accurately fitted by some analytic basic functions. To model accurately the 3D variation of the electron density in the IRI ionosphere over a fixed latitude and longitude range, the latitude and longitude variations at all altitudes need to be modelled.

Objectives

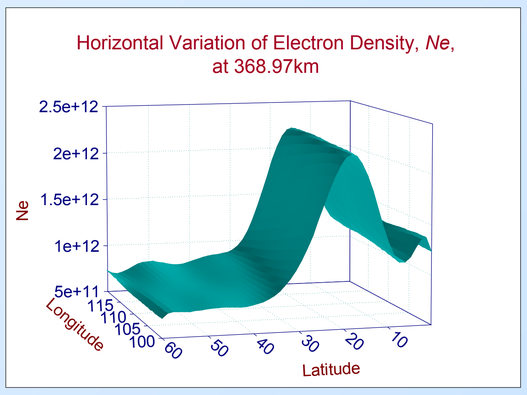
To develop a 3D ionospheric model which includes the variation of electron density in the IRI (or other 3D model) ionosphere over a desired latitude and longitude range that can be used in an accurate numerical ray-tracing program to determine precisely the effect of ionospheric horizontal gradients.

To trace transionospheric paths through the equatorial ionosphere, where the ionospheric horizontal gradient can be large, particularly in the region of the anomaly, and can cause refraction and significant differences in time delay compared with the transionospheric propagation in a homogenous ionosphere.

To investigate the error in using the Single Layer Model (SLM) to determine vertical TEC from slant TEC in the equatorial anomaly region and its dependence on time of a day, season, satellite and receiver location etc. To use additional parameters, employing the Modified-Single Layer Model (M-SLM), to obtain more accurate vertical TEC.

Modelling the 3D Ionosphere

The figure below shows the IRI model Ionosphere on 27 January 2003 at 13.30 LT near the height of the maximum of electron density which shows the large variation of electron density with latitude in the equatorial anomaly region.



Method Used

For the 3D model which was introduced in the numerical ray-tracing program, the electron density, *Ne*, at an altitude, *h*, latitude, θ and longitude, ϕ is given by;

$$Ne(h,\theta,\phi) = \sum_{i=1}^{i=n} Ne_{\max_i} \exp\left(-\left\{\frac{h-h_{\max_i}}{s_{\max_i}}\right\}^2\right) \frac{f_i(\theta,\phi)}{f_i(\theta_{ref},\phi_{ref})}$$

where;

$Ne_{\max_i}$ = maximum of electron density of an exponential layer at the reference latitude and longitude at the *ith* layer
$h_{\max_i}$ = height of the maximum of an exponential layer at the *ith* layer
$s_{\max_i}$ = semithickness parameter of an exponential layer at the *ith* layer
$f_i(\theta,\phi)$ = the function of electron density for a region of latitude and longitude at the *ith* layer
$f_i(\theta_{ref},\phi_{ref})$ = the function of electron density for the *ith* layer evaluated at the centre point of the modelled region of latitude and longitude (known as the reference location)
n = number of exponential layers used un the modelling

Fitting to the IRI profile was performed at a number of heights and the total electron density at any altitude (and latitude and longitude in the range) determined based on a sum of exponential layers [1] using the equation above.

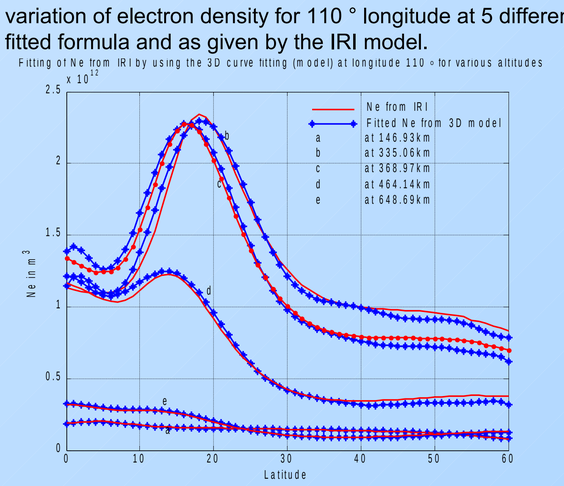
A large number of different equations were used to fit the IRI 3D electron density model and the formula below was found to be the optimum, taking into account also the constraint that all spatial derivatives must be continuous.

$$\begin{aligned} f(\theta,\phi) = {} & a + b\cos(\phi) + c\cos(\theta) + d\sin(\phi) + e\sin(\theta) + f\cos(2\phi) + g\cos(2\theta) + h\sin(2\phi) + i\sin(2\theta) + j\cos(3\phi) + k\cos(3\theta) + \\ & l\sin(3\phi) + m\sin(3\theta) + n\cos(4\phi) + o\cos(4\theta) + p\sin(4\phi) + q\sin(4\theta) + r\cos(5\phi) + s\cos(5\theta) + t\sin(5\phi) + u\sin(5\theta) + \\ & v\cos(\phi)\cos(\theta) + aa\cos(\phi)\sin(\theta) + ab\sin(\phi)\cos(\theta) + ac\cos(\phi)\cos(2\theta) + ad\cos(2\phi)\cos(\theta) + ae\cos(\phi)\sin(2\theta) + \\ & af\sin(2\phi)\cos(\theta) + ag\cos(\phi)\cos(3\theta) + ah\cos(3\phi)\cos(\theta) + ai\cos(\phi)\sin(3\theta) + aj\sin(3\phi)\cos(\theta) + ak\cos(\phi)\cos(4\theta) + \\ & al\cos(4\phi)\cos(\theta) + am\cos(\phi)\sin(4\theta) + an\sin(4\phi)\cos(\theta) + ao\sin(\phi)\sin(\theta) + ap\sin(\phi)\cos(2\phi) + aq\cos(2\phi)\sin(\theta) + \\ & ar\sin(\phi)\sin(2\theta) + as\sin(2\phi)\sin(\theta) + at\sin(\phi)\cos(3\theta) + au\cos(3\phi)\sin(\theta) + av\sin(\phi)\sin(3\theta) + ba\sin(3\phi)\sin(\theta) + \\ & bb\sin(\phi)\cos(4\theta) + bc\cos(4\phi)\sin(\theta) + bd\sin(\phi)\sin(4\theta) + be\sin(4\phi)\sin(\theta) + bf\cos(2\phi)\cos(2\theta) + bg\cos(2\phi)\sin(2\theta) + \\ & bh\sin(2\phi)\cos(2\theta) + bi\cos(2\phi)\cos(3\theta) + bj\cos(3\phi)\cos(2\theta) + bk\cos(2\phi)\sin(3\theta) + bl\sin(3\phi)\cos(2\theta) + bm\sin(2\phi)\sin(2\theta) + \\ & bn\sin(2\phi)\cos(3\theta) + bo\cos(3\phi)\sin(2\theta) + bp\sin(2\phi)\sin(3\theta) + bq\sin(3\phi)\sin(2\theta) \end{aligned}$$

where;

a,*b*,*c* … *bq* are the values of parameters which are obtained from 3D curve fitting software.

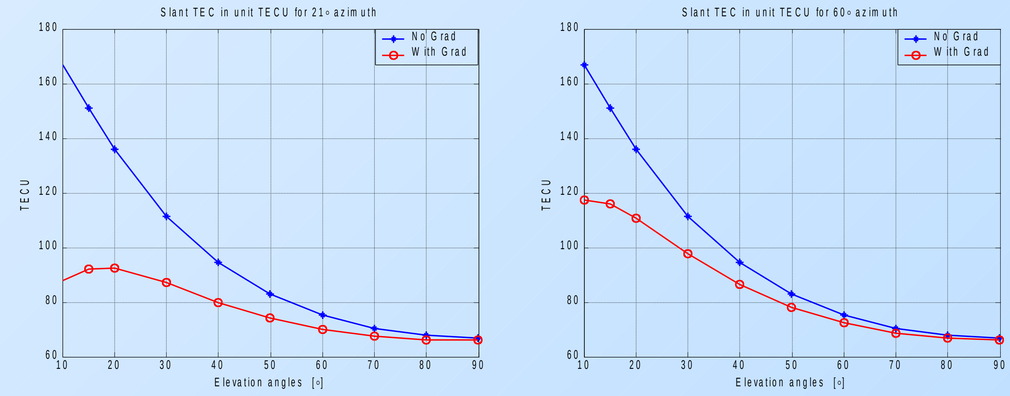
The figure shows the latitudinal variation of electron density for 110 ° longitude at 5 different altitudes as a function of latitude as determined from the fitted formula and as given by the IRI model.



Results and Analysis

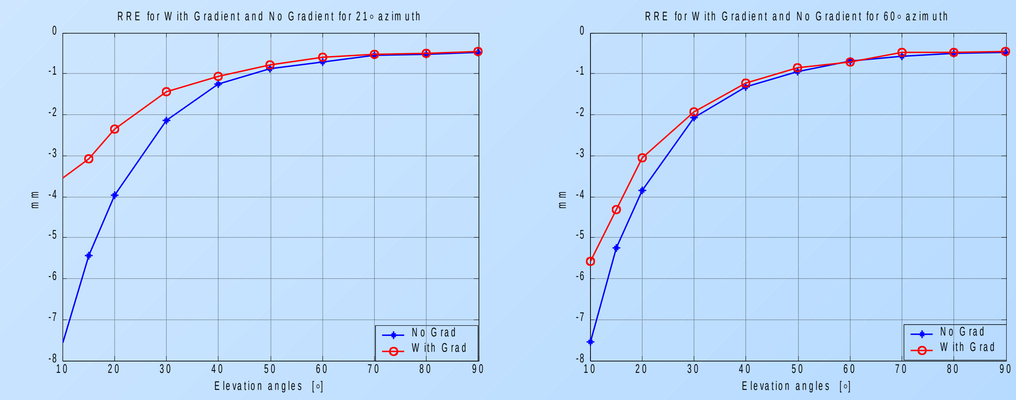
Some results have been obtained to characterize GPS ray paths using the numerical ray-tracing program for paths from a GPS satellite to an Earth Station at 15°N,110°E (the reference location) for both ionospheres with (3D model) and without horizontal gradients at two different azimuth angles, 21° and 60° from North.

Slant TEC



- The numerical ray-tracing with the new 3D model (with ionospheric gradient) shows a significant difference compared to the no gradient case. The slant TEC is less because of the lower electron density on the ray path at higher latitudes in the 3D model (as the electron density is fixed at the reference location values in the non-gradient case).
- The difference in slant TEC is larger for smaller azimuths because the ray travels further in the South-North direction where the gradient is largest.

The Residual Range Error, RRE



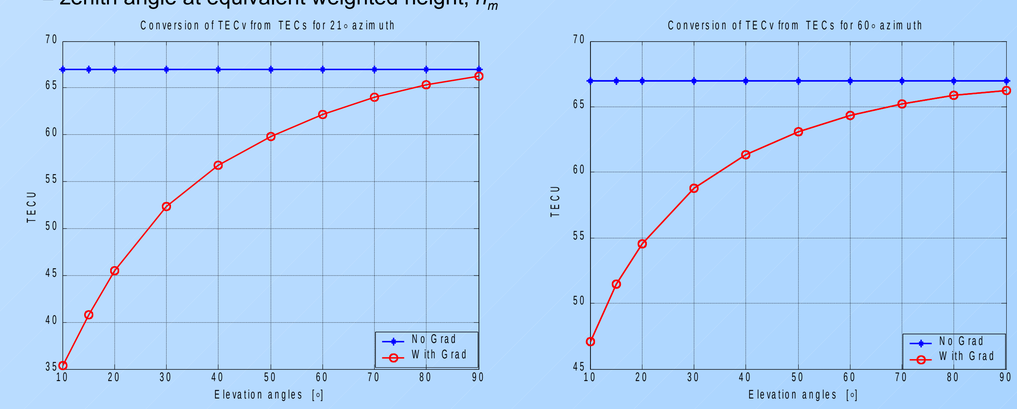
- Similar effects are seen in the RRE (Residual Range Error after dual frequency correction) as for the slant TEC above.

Slant TEC, TEC_s to Vertical TEC, TEC_v using M-SLM [2]

$$\sin\chi' = \frac{R_E}{R_E + h_m}\sin(a\chi) \qquad TEC_v = TEC_s\cos\chi'$$

where;

a = correction factor, taken here as 0.9782
R_E = mean Earth radius, 6370 km
h_m = equivalent weighted height, 438.1 km
χ' = zenith angle at the receiver site
χ = zenith angle at equivalent weighted height, h_m



- In the figure, one can see the significant effect the ionospheric horizontal gradients have on the determined vertical TEC compared to the no gradient case, when the vertical TEC is correctly determined for both azimuths.
- A correction factor '*a*' and an equivalent weighted height h_m =438.1km (instead of 350km in the Klobuchar approx.) are introduced in the model in order to get a more accurate transformation from TEC_s to TEC_v for the no gradient case.

Conclusions

- An accurate numerical ray-tracing model including the effect of ionospheric horizontal gradient in three dimensions (in latitude, longitude and altitude) has been constructed. This has been achieved using a 3D electron density model which has continuous spatial derivatives.
- The presence of the ionospheric horizontal gradient associated with the equatorial anomaly has been found to introduce significant effects on the group path delay of GPS signals, particularly at smaller azimuth angles.
- The importance of the introduction of an additional factor in the Modified-Single Layer Model (M-SLM) as well as the change of h_m in order to get an accurate transformation from slant to vertical TEC shows that the general SLM (Klobuchar approximation) is not accurate even for the no-gradient case.

References

[1] Hal J. Strangeways and Rigas T. Ioannides, 'Determination of Errors in Finding Vertical from Slant TEC due to Horizontal Gradients', Symposium of Atmospheric Remote Sensing Using Satellite Navigation System, Matera, Italy, 13 – 15 Oct, 2003, published on conference CD ROM 2004.

[2] S. Schaer, ' Mapping and Predicting the Earth's Ionosphere using the Global Positioning System', PhD Thesis, Astronomical Institute, University of Berne, 1999.