ABSTRACT

In the first we determine the dimension of Heliosphere (modulation region), radial diffusion coefficient and other parameters of convection-diffusion and drift mechanisms of cosmic ray (CR) long term variation in dependence of particles energy, level of solar activity (SA) and general solar magnetic field. This important information we obtain on the basis of CR and SA data in the past taking into account the theory of convection-diffusion and drift global modulation of galactic CR in the Heliosphere. By using these results and published regularly elsewhere predictions of expected SA variation we may made prediction of expected in near future long-term cosmic ray intensity variation. We introduce new nominations: integral multiplicity and coupling function for radiation dose inside aircraft caused by galactic CR. By the method of coupling functions we estimate the connection between CR intensity long-term variation and radiation hazard for aircrafts in dependence of altitude, geomagnetic cutoff rigidity and shielding inside aircraft. We show that by this way we may made monitoring and prediction of expected radiation hazard for any aircraft lines characterized by dependence from several parameters: altitude, cutoff rigidity, shielding. In this case become important also estimation of expected long-term changes in the planetary distribution of cutoff rigidities which also influenced on galactic CR intensity, and through CR – influenced on radiation hazard inside aircraft.

1. THE METHOD

The method, described below, takes into account that CR intensity observed on the Earth at a time \( t \) is caused by solar processes summarized for the long period started many months before \( t \). In paper [2] it was considered this method of using CR and SA data for solar cycles 19-22, also taking into account drift effects according to [3]. It was shown that including drift effects (as depending on the sign of solar polar magnetic field and determined by the difference of total CR modulation during \( A>0 \) and \( A<0 \) polarity cycles, and with the amplitude proportional to the value of tilt angles between the interplanetary neutral current sheet and the equatorial plane) is very important: it became possible to explain the great difference in time-lags between CR and SA in hysteresis phenomenon for even and odd solar cycles. But, are drift effects important for our case, when we consider situations near solar minima [2] showed that drift effects became negligible for some short period between two maxima of SA, when the drift effect does not change the sign; moreover, according to [3] this influence for high energy particles is expected especially small near minima of SA. Therefore in this paper we will
estimate the extent of the modulation region and of the residual CR modulation depending on the primary CR particle rigidity in the last solar minima without taking into account drift effects. This will be done in the context of a model for the global modulation of cosmic rays in the Heliosphere, taking into account time-lag processes in the interplanetary space relative to processes on the Sun using results of SA-CR hysteresis effects.

2. HYSTERESIS PHENOMENON AND MODEL OF COSMIC RAY GLOBAL MODULATION

It was shown in [10] that the time of propagation through the Heliosphere of particles with rigidity > 10 GV (to whom NM are sensitive) is no longer than one month (it will be shown again also below on the basis of new data). This time is at least about one order of magnitude smaller than the observed time-lag in the hysteresis phenomenon. This means that the hysteresis phenomenon on the basis of NM data can be considered as a quasi-stationary problem with parameters of CR propagation changing in time. In this case according to [12, 11]

\[ n(R_r,t)/n_o(R) \approx \exp \left( -a \int \frac{u(r,t)dr}{D_r(R_r,t)} \right), \]  

(1)

where \( n(R_r,t) \) is the differential rigidity CR density; \( n_o(R) \) is the differential rigidity density spectrum in the local interstellar medium out of the Heliosphere; \( a = 1.5 \); \( u(r,t) \) is the effective solar wind velocity (taking into account also shock waves and high speed solar wind streams); and \( D_r(R_r,t) \) is the effective radial diffusion coefficient in dependence of the distance \( r \) from the Sun of particles with rigidity \( R \) at the time \( t \). According to [8, 9] the connection between \( D_r(R_r,t) \) and solar activity can be described by the relation

\[ D_r(R_r,t) \propto r^\beta (W(t-r/u))^{-\alpha}, \]  

(2)

where \( W(t-r/u) \) is the sunspot number in the time \( t-r/u \). By the comparison with observation data it was determined in [8, 9] that parameter \( 0 \leq \beta \leq 1 \) and \( \alpha \approx 1/3 \) in the period of high solar activity \( W(t) \approx W_{\text{max}} \) and \( \alpha \approx 1 \) near solar minimum \( W(t) \approx W_{\text{min}} \). Here we suppose, in accordance with [6], that

\[ d(t) = 1/3 + (2/3)(1-W(t))/W_{\text{max}}, \]  

(3)

where \( W_{\text{max}} \) is the sunspot number in the maximum of solar activity cycle.

According to Eq. (1) the expected value of the natural logarithm of CR intensity global modulation at the Earth’s orbit, taking into account Eq. (2) and Eq. (3), will be

\[ \ln[n(R, X_o, \beta, R_x, t, \exp)] = A - B \times F(t, X_o, \beta, W(t-X)^{X_o}/X) \]  

(4)

where

\[ F(t, X_o, \beta, W(t-X)^{X_o}/X) = \]  

\[ X \int \frac{1/3 + 2/3(1-W(t)/W_{\text{max}})}{X_{\text{max}}^{X_o}/X} X^{-\beta} dX, \]

\[ X = r/u, X_{\text{max}} = 1AU/u, X_o = r_{o}/u, \]  

and

\[ n(R, X_o, \beta, R_x, t, \exp) \]  

is the expected galactic CR density at the Earth’s orbit in dependence of the values of parameters \( X_o \) and \( \beta \). Regression coefficients \( A(R, X_o, \beta, t_1, t_2) \) and \( B(R, X_o, \beta, t_1, t_2) \) can be determined by correlation between observed values \( \ln[n(R, X_o, t, \exp)] \) and the values of \( F(t, X_o, \beta, W(t-X)^{X_o}/X) \), calculated according to Eq. (5) for different values of \( X_o \) and \( \beta \) for the period of time between \( t_1 \) and \( t_2 \). In [5] three values of \( \beta = 0, 0.5, \) and 1 have been considered; it was shown that \( \beta = 1 \) strongly contradicts CR and SA observation data, and that \( \beta = 0 \) is the most reliable value. Therefore, we will consider here only this value.

3. DIMENSION OF THE MODULATION REGION NEAR SOLAR MINIMUM 1994-1996

We used monthly data of sunspot numbers and Climax NM data (USA, Colorado, N39, W106, \( H = 3400 \text{ m} \), \( R_c = 2.99 \text{ GV} \), as well as Huancayo (Peru, S12, W75, \( R_c = 12.92 \text{ GV} \), \( H = 3400 \text{ m} \)) or Haleakala (Hawaii, N20, W156, \( R_c = 12.91 \text{ GV} \), \( H = 3030 \text{ m} \)) NM data for the last solar minima (January 1994 – January 1997, \( W \leq 40 \)). We calculated correlation coefficients \( \rho(X_o) \) between the natural logarithm of observed and expected counting rate according to Eq. (5) in dependence of \( X_o \) and \( u = 1, 2, 3, \ldots 60 \text{ av. months} \left( X_o \right. \) is measured in units of av. month = 365.25/12 days, \( r_{o} \) in AU, and \( u \) in AU/av. month). We estimate \( X_{o,\text{max}} \) at which correlation coefficients \( \rho(X_o) \) reaches the maximum value. To determine \( X_{o,\text{max}} \) more exactly, we approximated the dependence of \( \rho \) on \( X_o \) in the vicinity of \( X_{o,\text{max}} \) by a parabolic function

\[ \rho(X_o) = aX_o^2 + bX_o + c. \]

In this case

\[ d\rho/dX_o = 2aX_o + b, \]  

and \( X_{o,\text{max}} = -b/2a \).
For Climax NM monthly data LN(CL1M) we approximated \( \rho(X_o) \) by
\[
\rho(X_o) = 0.000915X_o^2 - 0.03778X_o - 0.54937 \tag{6}
\]
with correlation coefficient 0.943±0.012, thus obtaining
\[
X_{o\text{max}} = 20.6 \pm 1.2 \text{ av. months}, \quad \rho_{\text{max}} = -0.939. \tag{7}
\]
For Huancayo/Haleakala NM monthly data LN(HU/HAL1M) we obtained
\[
\rho(X_o) = 0.000859X_o^2 - 0.030173X_o - 0.644715 \tag{8}
\]
with correlation coefficient 0.987±0.003, and
\[
X_{o\text{max}} = 17.6 \pm 0.5 \text{ av. months}, \quad \rho_{\text{max}} = -0.910. \tag{9}
\]
According to direct measurements on space probes the average solar wind speed for the period 1965-1990 near the Earth’s orbit at \( r = 1 \) AU was \( u_1 = 4.41 \times 10^7 \text{ cm/s} = 7.73 \text{ AU/av. month}. \) The function \( u(r) \) is determined by solar wind interactions with galactic CR and anomaly component of CR, with neutral atoms penetrating from interstellar space and others. According to calculations in [4] the change of solar wind velocity with the distance \( r \) from the Sun can be described approximately as
\[
u(r) \approx u_1(1 - b(r/r_{\text{tsw}})), \tag{10}
\]
where \( r_{\text{tsw}} \) is the distance to the terminal shock wave and parameter \( b = 0.13 \div 0.45 \) in dependence of sub-shock compression ratio and from injection efficiency of pickup protons. On the basis of Eq. (10) we can determine radius of CR modulation region \( r_{\text{mod}} \) from equation:
\[
X_{o\text{max}} = r_{\text{mod}} = \int_0^1 (u_1(1-b(r/r_{\text{tsw}})))^{-1} dr = -r_{\text{tsw}} \ln(-b + r_{\text{mod}}/r_{\text{tsw}})/b(u_1) \tag{11}
\]
from what follows
\[
r_{\text{mod}} = r_{\text{tsw}}(b + \exp(-X_{o\text{max}}b(u_1)/r_{\text{tsw}})). \tag{12}
\]
Let us assume that the radius of modulation region \( r_{\text{mod}} \) for Climax NM data (effective rigidity 10-15 GV) is about the same as radius of the Heliosphere \( r_{\text{tsw}} \). In this case from Eq. (12) at \( r_{\text{mod}} = r_{\text{tsw}} \) we obtain
\[
r_{\text{mod}} = -bu_1X_{o\text{max}}/\ln(1-b). \tag{13}
\]
For the average velocity of solar wind at the most reliable value of \( b \approx 0.3 \) we obtain from Eq. (13)
\[
u_{av} = -u_1b/\ln(1-b) = 0.84u_1, \tag{14}
\]
what for Climax NM gives \( r_{\text{mod}} = 134 \pm 8 \text{ AU}, \) and for Huancayo/Haleakala NM \( r_{\text{mod}} = 114 \pm 4 \text{ AU}. \)

4. ESTIMATION OF CORRELATION AND REGRESSION COEFFICIENTS

Determination of regression coefficients \( A \) and \( B \) in Eq. (4) makes it possible to determine the CR intensity outside of the modulation region, and the effective radial diffusion coefficient depending on the effective particle rigidity \( R \). The use of monthly data allows the determination of regression coefficients \( A \) and \( B \) only for integer values of \( X_o \). To increase the accuracy, we also use 11-month-moving averaged data. Therefore, for example, for LN(CL11M) we determined \( A \) and \( B \) for \( X_o = 20 \) (\( A = 8.367430, B = 0.006678 \)) and for \( X_o = 21 \) (\( A = 8.367825, B = 0.006285 \)), and then by interpolation for \( X_{o\text{max}} = 20.6 \). In the same way we determined \( A \) and \( B \) for LN(HU/HAL11M).

5. COSMIC RAY INTENSITY OUTSIDE THE HELIOSPHERE

The regression coefficient \( A \) in Eq. 4 according to Eq. 1 is
\[
A = \ln(n_o(R)), \tag{15}
\]
i.e. this coefficient determines the galactic CR intensity outside of the modulation region. In our case the coefficient \( A \) means the logarithm of the CR intensity outside the modulation region based on the Climax or by Huancayo/Haleakala NM, i.e. corresponds to primary CR particles with effective rigidities about 10-15 GV and 30-40 GV, respectively. We estimate that the accuracy in determining \( \ln(n_o(R)) \) for the Climax NM is ± 0.004 %, and for the Huancayo/Haleakala NM it is ± 0.008 %. In January 1994 LN(CL11M) = 8.31754, LN(HU/HAL11M) = 7.44217, that residual modulation (as modulation relative to the CR intensity out of the Heliosphere) was 5.014 ± 0.004% and 2.144 ± 0.008% for galactic CR with effective rigidity 10-15 GV and 30-40 GV. The residual modulation is almost inversely proportional to the effective rigidity of CR particles. Many scientists assume that for minimum SA detected by NM CR reaches an intensity very near to the intensity outside the Heliosphere. Let us check this. The maxima of LN(CL11M) and LN(HU/HAL11M) were reached in June and July 1997 with values 8.360387 and 7.46112 (minimal residual modulations of 0.361 ± 0.004% and 0.249 ± 0.008% for 10-15 and 30-40 GV particles). The obtained results show that even high energy CR particles (10-15 GV and 30-40 GV) inside the Heliosphere on the Earth’s orbit never reach the intensity out of the Heliosphere (in the interstellar...
6. PREDICTION OF COSMIC RAY VARIATIONS BY INTEGRAL F NEAR SA MINIMUM

As illustration, in Figure 1 are shown predicted by the integral F (calculated on the basis of monthly sunspot numbers \( W \) according to Eq. (5)) time variations and comparison with the observed natural logarithm of the month's average counting on Climax NM LN(CL11M) and for 11 months smoothed LN(CL11M). In this case we did not take into account the drift effects because according to [3] for high energy particles (for protons with energy much more than 1 GeV) near the SA minimum they are negligible in comparison with convection-diffusion modulation which does not depend from the sign of the solar general magnetic field. For Climax NM the correlation coefficient between predicted \( F \) and observed values of CR intensity LN(CL11M) was found equal to 0.993 ± 0.002. The same analysis for Huancayo/Haleakala NM gave correlation coefficient between predicted \( F \) and observed values of CR intensity LN(HU/HAL11M) equal to 0.970 ± 0.007.

7. FORECASTING OF CR INTENSITY DURING THE PERIOD OF SA INCREASING

In Section 6 we considered the forecasting of CR intensity near the minimum of SA when the drift effects are negligible. To demonstrate how can be taken into account the drift effects, let us consider, for example, the forecasting of CR intensity during the period of SA increasing in the onset of solar cycle during January 1996 – August 1999. In this case there are no information on the amplitude of drift modulation \( A_{dr} \), which is suggested proportional to the theoretically expected according to [3] and normalized to sunspot number \( W = 75 \). If the cycle is only started, we did not know \( A_{dr} \) for this cycle, but it is known type of cycle (odd or even) and we can use published predicted values of sunspot numbers for few years ahead. That let us use Eq. (4) and (5) for convection-diffusion modulation and average value of \( A_{dr} \) obtained for previous cycles 19-22 in [2]: \( A_{dr} \approx 2\% \) and 0.25% at \( W = 75 \) for Climax NM (effective rigidity of primary particles 10-15 GV) and Huancayo/Haleakala NM (35-45 GV) accordingly. Predicted CR intensity variations (separately expected convection-diffusion modulation and expected convection-diffusion + drift modulations) and observed CR long-term variation during 1996 - 2000 are shown in Figure 2 for Climax NM.

It can be seen that in this case the taking into account drift effects is sufficient. Correlation coefficient between predicted and observed cosmic radiation is found 0.988. For Huancayo/Haleakala NM with \( A_{dr} \approx 0.25\% \) at \( W = 75 \) the correlation coefficient is found 0.986.

8. ON THE RELATIVE ROLE OF DRIFT MODULATION IN MINIMA AND MAXIMA OF SOLAR ACTIVITY ON THE BASIS OF DATA FOR 1953 – 2000

Let us consider all period from 1953 to 2000 by taken into account drift effects during all solar cycles automatically according to described above methodic based on data of tilt angle and sunspot numbers. Based on data for 18 years (May 1976 - September 1993), we found that there are very good relation between \( T \) and \( W \); for 11 months smoothed data \( T = 0.349W + 13.5^\circ \) with correlation coefficient 0.955. We used 11 months smoothed data of \( W \) (shown in Figure 3) and determined the amplitude \( A_{dr} \) of drift effects as drift modulation at W11M = 75 (average value of W11M for 1953-1999). Or the information on reversal periods, we
used the following (ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA): August 1949 ± 9 months, December 1958 ± 12 months, December 1969 ± 8 months, March 1981 ± 5 months, and June 1991 ± 7 months. The final results for the period January 1953-November 2000 are shown in Figure 3.

Figure 3. CR data correction on drift effects in 1953-2000 (19-22 solar cycles and onset of 23 solar cycle): LN(CL11M) – observed natural logarithm of Climax neutron monitor counting rate smoothed for 11 months, LN(CL11M) DR2% – corrected Climax data on assumed drift effect according to burger and Potgieter (1999) with $A_{dr}=2\%$ at W11M = 75.

Interval between two horizontal lines is correspond variation of CR intensity on 5%. Smoothed sunspot numbers W11M are shown for comparison.

In Figure 3 are shown 5 minimums of solar activity, and in all minimums there are very good coincidence of corrected for drift effects CR intensity and observed CR intensity means that in minima of solar activity drift effects are not sufficient. From other hand, sufficient difference (few percents) we see in maximums of solar activity what means that in maxima of solar activity drift effects are important.

9. ON THE CONNECTION OF GALACTIC CR INTENSITY WITH DIFFERENTIAL AND INTEGRAL RADIATION DOSES INSIDE AIRCRAFT

Let us introduce some new definition: integral multiplicity $M_{rd}(R,S,h)$ for radiation dose from one primary CR particle with rigidity $R$ (in GV) inside the aircraft under shielding $S$ (in g/cm$^2$) at altitude determined by air pressure $h$ (also in g/cm$^2$). In this case the differential radiation dose per unit of time $I_{rd}(S,h,R_c,t)$ will be:

$$I_{rd}(S,h,R_c,t) = \frac{\infty}{R_c} D(R,t) M_{rd}(R,S,h) dR,$$  

where $D(R,t)$ is the differential primary rigidity spectrum of CR. Let us suppose that $D_{rd}(S,h,R_c,t)$ are measured in a broad interval of cutoff rigidities, and $D(R,t)$ is also known. In this case from Eq. 16 we obtain

$$\frac{\partial I_{rd}(S,h,R_c,t)}{\partial R_c} = -D(R_c,t) M_{rd}(R_c,S,h).$$  

From Eq. 17 follows

$$M_{rd}(R,S,h) = -\left(\frac{\partial I_{rd}(S,h,R_c,t)}{D(R_c,t) R_c} \right)_{R_c \rightarrow R}.$$  

The integral radiation dose $I_{rd}(S,h,R_c,t_1,t_2)$ during the flight from $t_1$ to $t_2$ will be determined by

$$I_{rd}(S,h,R_c,t_1,t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} I_{rd}(S,h,R_c,t) dt = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} D(R,t) M_{rd}(R,S,h(t)) dR.$$  

10. MAIN FACTORS DETERMINED THE VARIATION OF DIFFERENTIAL RADIATION DOSE WITH TIME

From Eq. 16 follows that

$$\delta D_{rd}(S,h,R_c,t) = \frac{\infty}{R_c} \delta D(R,t) M_{rd}(R,S,h) dR + \int D(R,t) M_{rd}(R,S,h) dR - D(R_c,t) M_{rd}(R_c,S,h)$$

Let us consider the relative variations of the differential radiation dose:

$$\frac{\partial D_{rd}(S,h,R_c,t)}{D_{rd}(S,h,R_c,t_0)} = \frac{\infty}{R_c} \frac{\delta D(R,t)}{D(R,t_0)} W_{rd}(R_c,R,S,h) dR + \int \frac{\delta M_{rd}(R,S,h)}{M_{rd}(R,S,h)} W_{rd} dR - W_{rd}(R_c,R,S,h)$$

where

$$W_{rd}(R_c,R,S,h) = \frac{D(R_c,t_0) M_{rd}(R,S,h)}{I_{rd}(S,h,R_c,t_0)}.$$
11. ANALYTICAL PRESENTATION OF RADIATION DOZE COUPLING FUNCTIONS FOR AIRCRAFTS

In many papers it was shown (see review in [1]) that any coupling function (for neutron monitors, for muon telescopes, for balloon and aircraft CR measurements, for CR measurements on balloons and satellites) can be presented in analytical form through the polar normalized coupling functions by the so called in literature Dorman function (introduced in [10])

\[ W_{om}(R) = a_m k_m R^{-k_m+1} \exp(-a_m R^{-k_m}), \] (23)

where parameters \( a_m \) and \( k_m \) depend from the air pressure \( h \) and the level of solar activity. Let us note that these functions, described by Eq. 23, are normalized: \( \int_0^{\infty} W_{om}(R) dR = 1 \) at any values of \( a_m \) and \( k_m \).

The same analytical form will be for the radiation doze coupling function for aircrafts \( W_{rd}(R_c, R, S, h) \), determined by the Eq. 22, but in this case parameters \( a_m \) and \( k_m \) will depend not only from the air pressure \( h \) and the level of solar activity, but also from shielding \( S \). The normalized coupling functions for point with cut-off rigidity \( R_c \), will be:

\[ W_{rd}(R_c, R, S, h) = a_R k_R R^{-(k_R+1)} \left[ 1 - a_R R^{-k_R} \right]^{-1} \exp(-a_R R^{-k_R}), \]

if \( R \geq R_c \); and \( W_{rd}(R_c, R, S, h) = 0 \), if \( R < R_c \). (24)

From Eq. 22 and Eq. 24 can be determined integral multiplicity \( M_{rd}(R, S, h) \) for aircraft radiation doze:

\[ M_{rd}(R, S, h) = \frac{\alpha_{rd}(S, h, R_c, t_R)}{\mathcal{D}(R, t_R)} a_R k_R R^{-(k_R+1)} \times \left[ 1 - a_R R^{-k_R} \right]^{-1} \exp(-a_R R^{-k_R}) \mid_{R \rightarrow R}. \] (25)

Eq. 18 and 25 show that on the basis of latitude surveys on different airplanes can be determined integral multiplicity \( M_{rd}(R, S, h) \) for radiation doze – an important characteristics for any aircraft.

12. MONITORING AND FORECASTING OF RADIATION DOZE INSIDE THE AIRCRAFT

By formulas in Sections 9 – 11 on the basis of ground CR measurements by neutron monitors and muon telescopes may be organized on line monitoring of radiation doze inside any aircraft in dependence of shielding \( S \), of the pressure (or altitude) \( h(t) \) and cutoff rigidity \( R_c(t) \) of the aircraft trajectory according to Eq. 19.

Moreover, on the basis of results obtained in Sections 1-8, can be made forecasting of expected CR intensity variation, and then by formulas in Sections 9 – 11 – forecasting of the expected radiation doze for any type of aircraft characterized by some shielding \( S \) and any flight trajectory characterized by parameters \( h(t) \) and cutoff rigidity \( R_c(t) \).

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