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THE SOLVING OF THE INVERSE PROBLEM IN COSMIC RAYS FOR SPACE WEATHER APPLICATIONS

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ABSTRACT

The observed energy spectrum of SEP and its change with time are determined by the energy spectrum in the source, by the time of SEP ejection into the solar wind and by the parameters of SEP propagation in the interplanetary space in dependence of particle energy. Here we will try to solve the inverse problem: on the basis of cosmic ray (CR) observations by the ground base detectors and detectors in the space to determine the energy spectrum of SEP in the source, the time of SEP ejection into the solar wind and the parameters of SEP propagation in the interplanetary space in dependence of particle energy. In general, this inverse problem is very complicated, and we suppose to solve it approximately step by step. In this paper we present the solution of the inverse problem in the frame of the simple model of isotropic diffusion (the first step). We suppose that after start of SEP event, the energy spectrum of SEP at different moments of time is determined with a good accuracy in a broad interval of energies. We show that after this can be determined the time of ejection, diffusion coefficient in the interplanetary space and energy spectrum in source of SEP. We show that for this necessary to

determine energy spectrum of SEP on the Earth at least in three different moments of time.

1. Observation data and inverse problems for isotropic diffusion, for anisotropic diffusion, and for kinetic description of solar CR propagation

It is well known that Solar Energetic Particle (SEP) events in the beginning stage are very anisotropic, especially during great events as in February 1956, July 1959, August 1972, September-October 1989, July 2000, January 2005, and many others (Dorman, M1957, M1963a,b, M1978; Dorman and Miroshnichenko, M1968; Miroshnichenko, M2001). To determine on the basis of experimental data the properties of the SEP source and parameters of propagation, i.e. to solve the inverse problem, is very difficult, and it needs data from many CR stations. By the procedure developed in Dorman and Zukerman (2003), Dorman, Pustil'nik, Sternlieb, and Zukerman, 2005; see review in Chapter 3 in Dorman, M2004), for each CR station the starting moment of SEP event can be automatically determined and then for different moments of time by the method of coupling functions to determine the energy spectrum of SEP out of the atmosphere above the individual CR station. As result we may obtain the planetary distribution of SEP intensity out of the atmosphere and then by taking into account the influence of geomagnetic field on particles trajectories – the SEP angle distribution out of the Earth's magnetosphere. By this way by using of the planetary net of CR stations with on-line registration in real time scale can be organized the continue on-line monitoring of great ground observed SEP events (Dorman, Pustil'nik, Sternlieb et al., 2004; Mavromichalaki, Yanke, Dorman et al., 2004).

In this paper we practically base on the two well established facts:

1) the time of particle acceleration on the Sun and injection into solar wind is very short in comparison with time of propagation, so it can be considered as delta-function from time;
2) the very anisotropic distribution of SEP with developing of the event in time after few scattering of energetic particles became near isotropic (well known examples of February 1956, September 1989 and many others).

This paper is the first step for solution of inverse problem in the theory of solar CR propagation by using only one on-line detector on the ground for high energy particles and one on-line detector on satellite for small energies. Therefore we will base here on the simplest model of generation (delta function in time and in space) and on the simplest model of propagation (isotropic diffusion). The second step will be based on anisotropic diffusion, and the third – on kinetic description of SEP propagation in the interplanetary space.

The observed energy spectrum of SEP and its change with time are determined by the energy spectrum in the source, by the time of SEP ejection into the solar wind and by the parameters of SEP propagation in the interplanetary space in dependence of particle energy. Here we will try to solve the inverse problem: on the basis of CR observations by the ground base detectors and detectors in the space to determine the energy spectrum of SEP in the source, the time of SEP ejection into the solar wind and the parameters of SEP propagation in the interplanetary space in dependence of particle energy. In general this inverse problem is very complicated, and we suppose to solve it approximately step by step. In this Section we present the solution of the inverse problem in the frame of the simple model of isotropic diffusion of solar CR (the first step). We suppose that after start of SEP event, the energy spectrum of SEP at different moments in time is determined with good accuracy in a broad interval of energies by the method of coupling functions (see in detail in Chapter 3 in Dorman, M2004). We show then that after this the time of ejection, diffusion coefficient in the interplanetary space and energy spectrum in source of SEP can be determined. This information, obtained on line on the basis of real-time scale data, may be useful also for radiation hazard forecasting.

2. The inverse problem for the case when diffusion coefficient depends only from particle rigidity

In this case the solution of isotropic diffusion for the pointing instantaneous source described by function

$$Q(R,r,t) = N_o(R)\delta(r)\delta(t) \tag{1}$$

will be

$$N(R,r,t) = N_o(R) \times \left[2\pi^{1/2}\left(\kappa(R)t\right)^{3/2}\right]^{-1} \times \exp\left(-\frac{r^2}{4\kappa(R)t}\right), \tag{2}$$

where r is the distance from the Sun, t is the time after ejection, $N_o(R)$ is the rigidity spectrum of total number of SEP at the source, and $\kappa(R)$ is the diffusion coefficient in the interplanetary space during SEP event. Let us suppose that at distance from the Sun $r = r_1 = 1\,\mathrm{AU}$ and at several moments of time t_i $(i = 1,\ 2,\ 3,...)$ after SEP ejection into solar wind the observed rigidity spectrum out of the Earth's atmosphere $N(R, r_1, t_i) \equiv N_i(R)$ are determined in high energy range on the basis of ground CR measurements by neutron monitors and muon telescopes (by using method of coupling functions, spectrographic and global spectrographic methods, see review in Dorman, M2004)) as well as determined directly in low energy range on the basis of satellite CR measurements. Let us suppose also that the UT time of ejection T_e as well as the diffusion coefficient $\kappa(R)$ and the SEP rigidity spectrum in source $N_o(R)$ are unknown. To solve the inverse problem, i.e. to determine these three unknown parameters, we need information on SEP rigidity spectrum $N_i(R)$ at least at three different moments of time T_1, T_2 and T_3 (in UT). In this case for these three moments of time after SEP ejection into solar wind we obtain:

$$t_1 = T_1 - T_e = x,\quad t_2 = T_2 - T_e = T_2 - T_1 + x,\quad t_3 = T_3 - T_e = T_3 - T_1 + x\,, \tag{3}$$

where T_2-T_1 and T_3-T_1 are known values and $x = T_1 - T_e$ is unknown value to be determined (because T_e is unknown). From three equations for t_1, t_2 and t_3 of the type of Eq. 2 by taking into account Eq. 3 and dividing one equation on other for excluding unknown parameter $N_o(R)$, we obtain two equations for determining unknown two parameters x and $\kappa(R)$:

$$\frac{T_2 - T_1}{x(T_2 - T_1 + x)} = -\frac{4\kappa(R)}{r_1^2} \times \ln\left\{\frac{N_1(R)}{N_2(R)}\left(x/(T_2 - T_1 + x)\right)^{3/2}\right\}, \tag{4}$$

$$\frac{T_3 - T_1}{x(T_3 - T_1 + x)} = -\frac{4\kappa(R)}{r_1^2} \times \ln\left\{\frac{N_1(R)}{N_3(R)}\left(x/(T_3 - T_1 + x)\right)^{3/2}\right\}. \tag{5}$$

To exclude unknown parameter $\kappa(R)$ let us divide Eq. 4 by Eq. 5; in this case we obtain equation for determining unknown $x = T_1 - T_e$:

$$x = \left[(T_2 - T_1)\Psi - (T_3 - T_1)\right]/(1 - \Psi), \tag{6}$$

where

$$\Psi=[(T_3-T_1)/(T_2-T_1)]\times\frac{\ln\left[N_1(R)(x/(T_2-T_1+x))^{3/2}/N_2(R)\right]}{\ln\left[N_1(R)(x/(T_3-T_1+x))^{3/2}/N_3(R)\right]}. \tag{7}$$

Eq. 6 can be solved by the iteration method: as a first approximation, we can use $x_1=T_1-T_e\approx 500$ sec which is the minimum time propagation of relativistic particles from the Sun to the Earth's orbit. Then, by Eq. 7 we determine $\Psi(x_1)$ and by Eq. 6 we determine the second approximation x_2. To put x_2 in Eq. 7 we compute $\Psi(x_2)$, and then by Eq. 6 we determine the third approximation x_3, and so on. After solving Eq. 6 and determining the time of ejection, we can compute very easily diffusion coefficient from Eq. 4 or Eq. 5:

$$\kappa(R)=-\frac{r_1^2(T_2-T_1)/4x(T_2-T_1+x)}{\ln\left\{\frac{N_1(R)}{N_2(R)}(x/(T_2-T_1+x))^{3/2}\right\}}=-\frac{r_1^2(T_3-T_1)/4x(T_3-T_1+x)}{\ln\left\{\frac{N_1(R)}{N_3(R)}(x/(T_3-T_1+x))^{3/2}\right\}}. \tag{8}$$

After determining the time of ejection and diffusion coefficient, it is easy to determine the SEP source spectrum:

$$\begin{aligned}N_o(R)&=2\pi^{1/2}N_1(R)\times(\kappa(R)x)^{3/2}\exp\left(r_1^2/(4\kappa(R)x)\right)\\&=2\pi^{1/2}N_2(R)\times(\kappa(R)(T_2-T_1+x))^{3/2}\exp\left(r_1^2/(4\kappa(R)(T_2-T_1+x))\right)\\&=2\pi^{1/2}N_3(R)\times(\kappa(R)(T_3-T_1+x))^{3/2}\exp\left(r_1^2/(4\kappa(R)(T_3-T_1+x))\right)\end{aligned} \tag{9}$$

3. The inverse problem for the case when diffusion coefficient depends from the particle rigidity and the distance to the Sun

Let us suppose, according to Parker (M1963), that the diffusion coefficient

$$\kappa(R,r)=\kappa_1(R)\times(r/r_1)^{\beta}. \tag{10}$$

In this case the solution of diffusion equation will be

$$N(R,r,t)=\frac{N_o(R)\times r_1^{3\beta/(2-\beta)}(\kappa_1(R)t)^{-3/(2-\beta)}}{(2-\beta)^{(4+\beta)/(2-\beta)}\Gamma(3/(2-\beta))}\times\exp\left(-\frac{r_1^{\beta}r^{2-\beta}}{(2-\beta)^2\kappa_1(R)t}\right), \tag{11}$$

where t is the time after SEP ejection into solar wind. So now we have four unknown parameters: time of SEP ejection into solar wind T_e, β, $\kappa_1(R)$, and $N_o(R)$. Let us assume that according to ground and satellite measurements at the distance $r=r_1=1$ AU from the Sun we know $N_1(R)$, $N_2(R)$, $N_3(R)$, $N_4(R)$ at UT times T_1, T_2, T_3, T_4. In this case

$$t_1=T_1-T_e=x,\quad t_2=T_2-T_1+x,\quad t_3=T_3-T_1+x,\quad t_4=T_4-T_1+x, \tag{2.34.12}$$

For each $N_i(R,r=r_1,T_i)$ we obtain from Eq. 2.34.11 and Eq. 2.34.12:

$$N_i(R, r = r_1, T_i) = \frac{N_o(R) \times r_1^{3\beta/(2-\beta)} (\kappa_1(R)(T_i - T_1 + x))^{-3/(2-\beta)}}{(2-\beta)^{(4+\beta)/(2-\beta)} \Gamma(3/(2-\beta))} \times \exp\left(-\frac{r_1^2 (2-\beta)^{-2}}{\kappa_1(R)(T_i - T_1 + x)}\right), \quad (13)$$

where i = 1, 2, 3, and 4. To determine x let us step by step exclude unknown parameters $N_o(R)$, $\kappa_1(R)$, and then β. In the first we exclude $N_o(R)$ by forming from four Eq. 13 for different i three equations for ratios

$$\frac{N_1(R, r = r_1, T_1)}{N_i(R, r = r_1, T_i)} = \left(\frac{x}{T_i - T_1 + x}\right)^{-3/(2-\beta)} \times \exp\left(-\frac{r_1^2}{(2-\beta)^2 \kappa_1(R)}\left(\frac{1}{x} - \frac{1}{T_i - T_1 + x}\right)\right), \quad (14)$$

where i = 2, 3, and 4. To exclude $\kappa_1(R)$ let us take logarithm from both parts of Eq. 2.34.14 and then divide one equation on another; as result we obtain following two equations:

$$\frac{\ln(N_1/N_2) + (3/(2-\beta))\ln(x/(T_2 - T_1 + x))}{\ln(N_1/N_3) + (3/(2-\beta))\ln(x/(T_3 - T_1 + x))} = \frac{(1/x) - (1/(T_2 - T_1 + x))}{(1/x) - (1/(T_3 - T_1 + x))}, \quad (15)$$

$$\frac{\ln(N_1/N_2) + (3/(2-\beta))\ln(x/(T_2 - T_1 + x))}{\ln(N_1/N_4) + (3/(2-\beta))\ln(x/(T_4 - T_1 + x))} = \frac{(1/x) - (1/(T_2 - T_1 + x))}{(1/x) - (1/(T_4 - T_1 + x))}. \quad (16)$$

After excluding from Eq. 15 and Eq. 16 unknown parameter β, we obtain equation for determining x:

$$x^2(a_1 a_2 - a_3 a_4) + xd(a_1 b_2 + b_1 a_2 - a_3 b_4 - b_3 a_4) + d^2(b_1 b_2 - b_3 b_4) = 0, \quad (17)$$

where

$$d = (T_2 - T_1)(T_3 - T_1)(T_4 - T_1), \quad (18)$$

$$a_1 = (T_2 - T_1)(T_4 - T_1)\ln(N_1/N_3) - (T_3 - T_1)(T_4 - T_1)\ln(N_1/N_2), \quad (19)$$

$$a_2 = (T_3 - T_1)(T_4 - T_1)\ln(x/(T_2 - T_1 + x)) - (T_2 - T_1)(T_3 - T_1)\ln(x/(T_4 - T_1 + x)), \quad (20)$$

$$a_3 = (T_2 - T_1)(T_3 - T_1)\ln(N_1/N_4) - (T_3 - T_1)(T_4 - T_1)\ln(N_1/N_2), \quad (21)$$

$$a_4 = (T_3 - T_1)(T_4 - T_1)\ln(x/(T_2 - T_1 + x)) - (T_2 - T_1)(T_4 - T_1)\ln(x/(T_3 - T_1 + x)), \quad (22)$$

$$b_1 = \ln(N_1/N_3) - \ln(N_1/N_2), \;\; b_2 = \ln(x/(T_2 - T_1 + x)) - \ln(x/(T_4 - T_1 + x)), \quad (23)$$

$$b_3 = \ln(N_1/N_4) - \ln(N_1/N_2), \;\; b_4 = \ln(x/(T_2 - T_1 + x)) - \ln(x/(T_3 - T_1 + x)). \quad (24)$$

As it can be seen from Eq. 20 and Eq. 22-24, coefficients a_2, a_4, b_2, b_4 very weekly (as logarithm) depend from x. Therefore Eq. 17 we solve by iteration method, as above we solved Eq. 6: as a first approximation, we use $x_1 = T_1 - T_e \approx 500$ sec (which is the minimum time propagation of relativistic particles from the Sun to the Earth's orbit). Then by Eq. 20 and Eq. 22-24 we determine $a_2(x_1)$, $a_4(x_1)$, $b_2(x_1)$, $b_4(x_1)$ and by Eq. 17 we determine the second approximation x_2, and so on. After determining *x*, i.e. according Eq. 12 determining t_1, t_2, t_3, t_4, the final solutions for β, $\kappa_1(R)$, and $N_o(R)$ can be found. Unknown parameter β in Eq. 10 we determine from Eq. 15 and Eq. 16:

$$\beta = 2 - 3\left[(\ln(t_2/t_1)) - \frac{t_3(t_2-t_1)}{t_2(t_3-t_1)}\ln(t_3/t_1)\right] \times \left[(\ln(N_1/N_2)) - \frac{t_3(t_2-t_1)}{t_2(t_3-t_1)}\ln(N_1/N_3)\right]^{-1}. \quad (25)$$

Then we determine unknown parameter $\kappa_1(R)$ in Eq. 10 from Eq. 14:

$$\kappa_1(R) = \frac{r_1^2\left(t_1^{-1} - t_2^{-1}\right)}{3(2-\beta)\ln(t_2/t_1) - (2-\beta)^2\ln(N_1/N_2)} = \frac{r_1^2\left(t_1^{-1} - t_3^{-1}\right)}{3(2-\beta)\ln(t_3/t_1) - (2-\beta)^2\ln(N_1/N_3)}. \quad (26)$$

After determining parameters β and $\kappa_1(R)$ we can determine the last parameter $N_o(R)$ from Eq. 13:

$$N_o(R) = N_i(2-\beta)^{(4+\beta)/(2-\beta)}\Gamma(3/(2-\beta))r_1^{-3\beta/(2-\beta)}(\kappa_1(R)t_i)^{3/(2-\beta)} \times \exp\left(\frac{r_1^2}{(2-\beta)^2\kappa_1(R)t_i}\right) \quad (27)$$

where index i = 1, 2 or 3.

Above we show that for some simple model of SEP propagation is possible to solve inverse problem based on ground and satellite measurements at the beginning of the event. Obtained results we used in the method of great radiation hazard forecasting based on on-line CR one-minute ground and satellite data (Dorman, Iucci, Murat et al., 2005).

Let us note that described solutions of inverse problem may be partly useful for solving more complicated inverse problems in case of SEP propagation described by anisotropic diffusion and by kinetic equation.

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