

Analytical Model for Galactic and Solar Cosmic Ray Ionization in the Planetary Ionospheres and Atmospheres

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Abstract

An analytical model for cosmic ray (CR) ionization in the terrestrial lower ionosphere and middle atmosphere is developed in this work. For this purpose the ionization losses (dE/dh) according the Bohr-Bethe-Bloch formula for the energetic charged particles are approximated in five different energy intervals similarly to Dorman (*Cosmic Rays in the Earth's Atmosphere and Underground*. Dordrecht, Kluwer Acad. Publ. 2004), but a few precision corrections are involved. More accurate expressions for energy decrease of particles $E(h)$ and electron production rate profiles $q(h)$ in the ionosphere and atmosphere are derived. The obtained formulas allow comparatively easy computer programming. The integrand in $q(h)$ gives the possibility for application of adequate numerical methods – such as Romberg method, or Gauss quadrature, for the solution of the mathematical problem. On this way the process of interaction of cosmic ray particles with the upper, middle and lower atmosphere will be described much more realistically. Here we enlarge significantly the energetic interval (5 - 5000 GeV) of the penetrating cosmic rays at their interaction with the atmospheres and ionospheres of the planets, satellites and comets. We introduce new approximation formula for the ionization losses (dE/dh) of the energetic charged particles in the energy interval (5 - 5000 GeV). Expressions are derived for cosmic ray protons, helium nuclei with charge $Z = 2$ (which are 12.6% from all CR nuclei), and nuclei with charge Z (CR group of nuclei L, M, H, VH, and SH). The calculations show, that independently on the domination of the protons and helium nuclei (approximately 98.5% from all particles) they create the same ionization as the ionization by multicharge ($Z > 2$) particles, which are about 1.5% from all CR particles. The explanation of this fact is that the multicharge nuclei have Z^2 greater ionization capability then the protons.

Introduction. In the paper [1] analytical model for cosmic ray proton ionization in the lower ionosphere and middle atmosphere was developed. Our model is near to Dorman analytical approach [2-5], which „describe the formation of ionization by protons and nuclei of different Z of solar and galactic cosmic rays (CR) by taking into account electron capturing and energy change of incident particles down to thermal energies kT ” [2-5]. The proposed model in [1] is not only approach and it gives some more precise specifications of the Bohr - Bethe - Bloch formula for the ionization losses in the five energetic intervals, which are considered. The experimental data confirm this new approximation [6-10]. In [1] it is applied for calculation of the energy decrease laws of the penetrating particles (in the case of protons) through the substance of the atmosphere for the intervals 0.15-200 MeV, 5-100 GeV and the electron production rate for cosmic ray protons in the interval 0.15-200 MeV, similarly to [2-5]. In [1] are also derived the respective atmospheric energy boundaries, which limit the intervals of the ionization losses function. The model of the electron production rate is an integral over the energy - Eqs.(13, 15, 17) [1]. Its solution is obtained as a sum of some subintegrals, which correspond to the boundaries of the energy decrease laws of the penetrating particles. The ionization path is calculated as an integral of the reverse value of the

ionization losses function over the energy - Eq.(3) [1]. Some characteristic ionization paths are found, which determine the main regions of the ionization losses function. The one-dimensional case is considered, i.e. in the model it is accepted, that the cosmic ray protons penetrate vertically towards the Earth surface.

One detailed analytical approach was proposed by Dorman [2]. He accepts five energetic intervals with different power values. In the present work more accurate approximations of the Bohr-Bethe-Bloch formula will be given. More precise expressions for energy decrease $E(h)$ and ionization q ($\text{cm}^{-3} \cdot \text{s}^{-1}$) in the atmosphere also will be derived. For this purpose the following formula for ionization path L will be used [1]:

$$L(h) = L(E) + \int_{E(h)}^E \frac{dE}{\rho \left(\frac{dE}{dh} \right)}$$

where E and $L(E)$ are the initial energy and ionization path, $E(h)$ and $L(h)$ are the energy and ionization path at altitude h .

Ionization losses. The ionization energy losses of cosmic ray particles can be approximated in the energy range E from kT to 5000 GeV as (in units $\text{MeV} \cdot \text{g}^{-1} \cdot \text{cm}^2$) [1]:

$$-\frac{1}{\rho} \frac{dE}{dh} = \begin{cases} 2.57 \times 10^3 E^{0.5} & \text{if } kT \leq E \leq 0.15 \text{ MeV} \\ 231 E^{-0.77} & \text{if } 0.15 \leq E \leq 200 \text{ MeV} \\ 68 E^{-0.53} & \text{if } 200 \leq E \leq 850 \text{ MeV} \\ 1.91 & \text{if } 850 \leq E \leq 5 \times 10^3 \text{ MeV} \\ 0.66 E^{0.123} & \text{if } 5 \times 10^3 \leq E \leq 5 \times 10^6 \text{ MeV} \end{cases}$$

Case for low energetic protons. As in the ionosphere the low energy particles play the most important role, let us first consider the case, where the initial proton energy at the atmosphere boundary is $0.15 \text{ MeV} < E_0 \leq 200 \text{ MeV}$. In this energy range is situated the bigger part of the solar cosmic rays. On the base of the equations we receive the dependence of the proton energy on h :

$$(5) \quad E(E_0, h) = \begin{cases} (E_0^{1.77} - 231 \times 1.77 h)^{1.77} & \text{if } h \leq h_4 \\ \left[(0.15)^{1/2} - 1.285 \times 10^3 (h - h_4) \right]^2 & \text{if } h_4 \leq h \leq h_5 \\ kT & \text{if } h \geq h_5 \end{cases}$$

where

$$h_4(E_0) = \frac{E_0^{1.77} - (0.15)^{1.77}}{231 \times 1.77} \text{ g} \cdot \text{cm}^{-2}$$

is the atmospheric depth, at which the proton energy equals $E = 0.15 \text{ MeV}$;

$$h_5(E_0) = 7,782 \times 10^{-4} \left[(0.15)^{1/2} - (kT)^{1/2} \right] + h_4(E_0) \text{ g} \cdot \text{cm}^{-2}$$

is the atmospheric depth, where the proton energy decreases until $E = kT$.

Case for higher energies. Similar expressions for $E(E_0, h)$ can be obtained if we suppose, that $5 \times 10^3 \text{ MeV} < E_0 < 5 \times 10^6 \text{ MeV}$. With these energies are included the galactic as also the solar cosmic rays. In this case is obtained:

$$E(E_0, h) = \begin{cases} (E_0^{0.877} - 0.877 \times 0.66 \times h)^{1/0.877} & \text{if } h \leq h_1 \\ 5 \times 10^3 - 1.91(h - h_1) & \text{if } h_1 \leq h \leq h_2 \\ [850^{1.53} - 68 \times 1.53(h - h_2)]^{1/1.53} & \text{if } h_2 \leq h \leq h_3 \\ [200^{1.77} - 231 \times 1.77(h - h_3)]^{1/1.77} & \text{if } h_3 \leq h \leq h_4 \\ [(0.15)^{1/2} - 1.285 \times 10^3(h - h_4)]^2 & \text{if } h_4 \leq h \leq h_5 \\ kT & \text{if } h \geq h_5 \end{cases}$$

where

$$h_1(E_0) = \frac{E_0^{0.877} - (5 \times 10^3)^{0.877}}{0.877 \times 0.66} \text{ g.cm}^{-2},$$

is the atmospheric depth, by which the proton energy is $E = 5 \times 10^3 \text{ MeV}$;

$$h_2(E_0) = (5 \times 10^3 - 850) / 1.91 + h_1(E_0) \text{ g.cm}^{-2},$$

is the atmospheric depth, by which the proton energy is $E = 850 \text{ MeV}$;

$$h_3(E_0) = (850^{1.53} - 200^{1.53}) / (68 \times 1.53) + h_2(E_0) \text{ g.cm}^{-2},$$

is the atmospheric depth, by which the proton energy is $E = 200 \text{ MeV}$;

$$h_4(E_0) = \frac{200^{1.77} - (0.15)^{1.77}}{231 \times 1.77} + h_3(E_0) \text{ g.cm}^{-2},$$

is the atmospheric depth, by which the proton energy is $E = 0.15 \text{ MeV}$. The depth $h_5(E_0)$ at which the proton energy reaches $E = kT$ will be determined by the presented equations.

Electron production rates by low energetic protons ($Z = 1$). The ionization profiles $q_1(h)$ by the primary proton ($Z = 1$) spectrum $D_1(E_0)$ will be:

$$q_1(h) = \frac{\rho(h)}{Q} \int_{E_{\min}}^{\infty} D_1(E_0) \left(-\frac{dE(E_0, h)}{dh} \right) dE_0$$

where $\rho(h)$ is the atmosphere density. Q is the energy required for creation of one electron – ion pair (for air $Q = 3.5 \times 10^{-5} \text{ MeV}$). E_{\min} is the minimal energy in proton differential spectrum, which is determined by geomagnetic cut-off rigidity, electric field cut-off or atmospheric cut-off.

Initially let us consider the most simple $D_1(E_0)$ for energies below 200 MeV but above 0.15 MeV (the ionization losses maximum). This case is significant for the upper and middle atmosphere. Here there are three possible cases:

Case for consideration: $D_1(E_0)$ for $0.15 < E_0 \leq 200 \text{ MeV}$

Case 1. When $200 > E_{\min} \geq E_1(h)$, where

$$E_1(h) = [231 \times 1.77h + (0.15)^{1.77}]^{1/1.77}$$

we obtain the following expression for the electron production rates

$$q_1(h) = \frac{\rho(h)}{Q} \int_{E_{\min}}^{200} D_1(E_0) \times 231 \times (E_0^{1.77} - 231 \times 1.77h)^{-0.77/1.77} dE_0$$

Case 2. When $200 > E_1(h) \geq E_{\min} \geq E_2(h)$, where

$$E_2(h) = [0.035 + 408.87(h - 7.782 \times 10^4(0.387 - (kT)^{0.5}))]^{1/1.77}$$

we obtain in this case

$$q_1(h) = 2.57 \times 10^3 \times \frac{\rho(h)}{Q} \int_{E_{\min}}^{E_1(h)} D_1(E_0) \times \left[0.387 + 1.285 \times 10^3 \times \left(\frac{E_0^{1.77} - 0.035}{231 \times 1.77} - h \right) \right] dE_0 +$$

$$+ \frac{\rho(h)}{Q} \int_{E_1(h)}^{200} D_1(E_0) \times 231 \times [E_0^{1.77} - 231 \times 1.77h]^{-0.77/1.77} dE_0$$

Case 3. When $E_{\min} \leq E_2(h)$. In this case the integral for ionization rates coincides with the expression above, but the first part is solved with lower boundary $E_2(h)$ instead of E_{\min} , i.e.

$$q_1(h) = 2.57 \times 10^3 \times \frac{\rho(h)}{Q} \int_{E_2(h)}^{E_1(h)} D_1(E_0) \times \left[0.387 + 1.285 \times 10^3 \times \left(\frac{E_0^{1.77} - 0.035}{231 \times 1.77} - h \right) \right] dE_0 +$$

$$+ \frac{\rho(h)}{Q} \int_{E_1(h)}^{200} D_1(E_0) \times 231 \times [E_0^{1.77} - 231 \times 1.77h]^{-0.77/1.77} dE_0$$

Analytical Model for Cosmic Ray Proton Ionization in the Energy Interval 200-850 MeV. It is assumed that the initial energy E_0 for cosmic ray proton ionization is in the interval: $200 \leq E_0 \leq 850$ MeV. The corresponding energy losses function is defined in three energy intervals as follows:

$$-\frac{1}{\rho} \frac{dE}{dh} = \begin{cases} 68 \times E^{-0.53} & \text{if } 200 \leq E \leq 850 \text{ MeV} \\ 231 \times E^{-0.77} & \text{if } 0.15 \leq E \leq 200 \text{ MeV} \\ 2.57 \times 10^3 E^{0.5} & \text{if } kT \leq E \leq 0.15 \text{ MeV} \end{cases}$$

where E is the kinetic energy of the protons in MeV. The energy decrease law in these intervals down to the thermal energy kT in dependence on the traveling path of substance h (g.cm⁻²) is

$$E(E_0, h) = \begin{cases} [E_0^{1.53} - 68 \times 1.53h]^{1/1.53} & \text{if } h \leq h_3 \\ [200^{1.77} - 231 \times 1.77(h - h_3)]^{1/1.77} & \text{if } h_3 \leq h \leq h_4 \\ [0.15^{1/2} - 1.285 \times 10^3(h - h_4)]^2 & \text{if } h_4 \leq h \leq h_5 \\ kT & \text{if } h \geq h_5 \end{cases}$$

The traveling path, which limits the upper energy interval (200-850 MeV) is derived:

$$h_3(E_0) = \frac{E_0^{1.53} - 200^{1.53}}{68 \times 1.53} \text{ g.cm}^{-2}$$

The next energy interval (0.15-200 MeV) is bounded by traveling path with value:

$$h_4(E_0) = h_3(E_0) + \frac{200^{1.77} - 0.15^{1.77}}{231 \times 1.77} \text{ g.cm}^{-2}$$

The thermal energy of the particles is reached for traveling path:

$$h_5(E_0) = h_4(E_0) + 7.782 \times 10^{-4} \left[(0.15)^{0.5} - (kT)^{0.5} \right] \text{ g.cm}^{-2}$$

The following four cases for calculation of the electron production rate correspond to energy intervals of the ionization losses function. The boundary E_{\min} is the minimal energy in proton spectrum, determined by geomagnetic cut-off or by some other reason [2]. It is assumed, that the energy for production of 1 electron - ion pair is $Q = 35$ eV and $\rho(h)$ is the neutral density.

We will consider 4 cases for the respective energy intervals. We take the lower energy value of the corresponding interval and solve towards E_0 the relations for the fixed current h .

Case 1: $E_1(h) \leq E_{\min} < 850$ MeV

The energy boundary for the lower limit of integration in the first case is:

$$E_1(h) = \left(200^{1.53} + 68 \times 1.53h \right)^{1/1.53}$$

The respective electron production rate is:

$$q_1(h) = \frac{\rho(h)}{Q} \int_{E_{\min}}^{850} D_1(E_0) \times 68 \times \left(E_0^{1.53} - 68 \times 1.53h \right)^{-0.53/1.53} dE_0$$

Case 2: $E_2(h) \leq E_{\min} < E_1(h) < 850$ MeV

The energy boundary for the lower limit of integration in the second case is

$$E_2(h) = \left[200^{1.53} + 68 \times 1.53 \left(h - \frac{200^{1.77} - 0.15^{1.77}}{231 \times 1.77} \right) \right]^{1/1.53}$$

The electron production rate is a sum of two integrals, for the corresponding energy intervals in the ionization losses function:

$$q_1(h) = \frac{\rho(h)}{Q} \int_{E_{\min}}^{E_1(h)} D_1(E_0) \times 231 \times \left[200^{1.77} + 231 \times 1.77 \left(\frac{E_0^{1.53} - 200^{1.53}}{68 \times 1.53} - h \right) \right]^{-0.77/1.77} dE_0 +$$

$$+ \frac{\rho(h)}{Q} \int_{E_1(h)}^{850} D_1(E_0) \times 68 \times \left[E_0^{1.53} - 68 \times 1.53h \right]^{-\frac{0.53}{1.53}} dE_0$$

Case 3: $E_3(h) \leq E_{\min} < E_2(h) < E_1(h) < 850$ MeV

The atmospheric boundary for the energy limit in the third case is

$$E_3(h) = \left\{ 200^{1.53} + 68 \times 1.53 \left[h - \frac{200^{1.77} - 0.15^{1.77}}{231 \times 1.77} - 7.782 \times 10^{-4} \times (0.15^{1/2} - (kT)^{1/2}) \right] \right\}^{1/1.53}$$

Now the three integrals are summed up for calculation of the electron production rate:

$$\begin{aligned} q_1(h) = & \frac{\rho(h)}{Q} \int_{E_{\min}}^{E_2(h)} D_1(E_0) \times 2.57 \times 10^3 \times \\ & \times \left[0.15^{0.5} - 1.285 \times 10^3 \left(h - \frac{E_0^{1.53} - 200^{1.53}}{68 \times 1.53} - \frac{200^{1.77} - 0.15^{1.77}}{231 \times 1.77} \right) \right]^{2/2} dE_0 + \\ & + \frac{\rho(h)}{Q} \int_{E_2(h)}^{E_1(h)} D_1(E_0) \times 231 \times \left[200^{1.77} - 231 \times 1.77 \times \right. \\ & \times \left. \left(h - \frac{E_0^{1.53} - 200^{1.53}}{68 \times 1.53} \right) \right]^{\frac{0.77}{1.77}} dE_0 + \\ & + \frac{\rho(h)}{Q} \int_{E_1(h)}^{850} D_1(E_0) \times 68 \times \left[E_0^{1.53} - 68 \times 1.53 h \right]^{\frac{0.53}{1.53}} dE_0 \end{aligned}$$

Case 4: If $E_{\min} \leq E_3(h) \rightarrow E_{\min} = E_3(h)$

In this case the expression which corresponds to the thermal energy boundary limits the first integral for calculation of the electron production rate. The other two integrals correspond to these in case 3.

Analytical Model for Cosmic Ray Proton Ionization in the Energy Interval 850-5000 MeV. The further considerations suggest the initial energy of the differential spectrum for protons of cosmic rays to be in the energy interval 850-5000 MeV. Now the ionization losses function is defined in four basic energetic intervals as follows:

$$\frac{1}{\rho} \frac{dE}{dh} = \begin{cases} 1.91 & \text{if } 850 \leq E \leq 5 \times 10^3 \text{ MeV} \\ 68 \times E^{-0.53} & \text{if } 200 \leq E \leq 850 \text{ MeV} \\ 231 \times E^{-0.77} & \text{if } 0.15 \leq E \leq 200 \text{ MeV} \\ 2.57 \times 10^3 E^{1/2} & \text{if } kT \leq E \leq 0.15 \text{ MeV} \end{cases}$$

The corresponding energy decrease laws for it are stated with the equations below:

$$E(E_0, h) = \begin{cases} E_0 - 1.9h & \text{if } 0 \leq h \leq h_2 \\ \left[850^{1.53} - 68 \times 1.53(h - h_2) \right]^{1/1.53} & \text{if } h_2 \leq h \leq h_3 \\ \left[200^{1.77} - 231 \times 1.77(h - h_3) \right]^{1/1.77} & \text{if } h_3 \leq h \leq h_4 \\ \left[0.15^{1/2} - 1.285 \times 10^3(h - h_4) \right]^2 & \text{if } h_4 \leq h \leq h_5 \\ kT & \text{if } h \geq h_5 \end{cases}$$

The next equation shows the traveling path of substance $h_2(E_0)$ for particles inside the energy interval for cosmic ray proton ionization: $850 \leq E_0 \leq 5000 \text{ MeV}$

$$h_2(E_0) = \frac{E_0 - 850}{1.91} \text{ g.cm}^{-2}$$

$h_3(E_0)$, $h_4(E_0)$, $h_5(E_0)$ are the traveling paths for the corresponding energetic intervals.

$$h_3(E_0) = \frac{850^{1.53} - 200^{1.53}}{68 \times 1.53} + h_2(E_0) \text{ g.cm}^{-2}$$

$$h_4(E_0) = \frac{200^{1.77} - 0.15^{1.77}}{231 \times 1.77} + h_3(E_0) \text{ g.cm}^{-2}$$

$$h_5(E_0) = \frac{0.15^{1/2} - (kT)^{1/2}}{1.285 \times 10^3} + h_4(E_0) \text{ g.cm}^{-2}$$

The next five cases of electron production rate calculation in the different energetic intervals of the ionization losses function are stated with account of the respective energy boundaries (atmospheric cut-offs), which define the limits of integration. They are obtained from the traveling paths of substance, which are solved towards E_0 . These calculations are similar to the electron production rate evaluations, which were already considered. The difference is in the energy intervals, which are introduced. One more interval is included here - with higher energy than that of the intervals, formulated above.

The above mentioned relations are solved towards E_0 for the fixed current h . On this way are obtained the parameters $E_i(h)$, $i=1, \dots, 4$ in the following cases 1 – 5.

Case 1: $E_1(h) \leq E_{\min} < 5000 \text{ MeV}$

The energy limit of integration boundary for the first interval is:

$$E_1(h) = 850 + 1.91 \times h$$

The corresponding electron production rate is:

$$q_1(h) = \frac{\rho(h)}{Q} \int_{E_{\min}}^{5000} D_1(E_0) \times 1.91 dE_0$$

Case 2: $E_2(h) \leq E_{\min} < E_1(h) < 5000 \text{ MeV}$

$E_2(h)$ is obtained from (14) solving towards E_0 :

$$E_2(h) = 850 + 1.91 \left(h - \frac{850^{1.53} - 200^{1.53}}{68 \times 1.53} \right)$$

The corresponding electron production rate is summed over two energy intervals:

$$q_1(h) = \frac{\rho(h)}{Q} \int_{E_{\min}}^{E_1(h)} D_1(E_0) \times 68 \times \left[850^{1.53} + 68 \times 1.53 \left(\frac{E_0 - 850}{1.91} - h \right) \right]^{\frac{0.53}{1.53}} dE_0 +$$

$$+ \frac{\rho(h)}{Q} \int_{E_1(h)}^{5000} D_1(E_0) \times 1.91 dE_0$$

Case 3: $E_3(h) \leq E_{\min} < E_2(h) < E_1(h) < 5000 \text{ MeV}$

$E_3(h)$ is obtained as follows:

$$E_3(h) = 850 + 1.91 \left(h - \frac{200^{1.77} - 0.15^{1.77}}{231 \times 1.77} - \frac{850^{1.53} - 200^{1.53}}{68 \times 1.53} \right)$$

The corresponding electron production rate is summed over three energy intervals:

$$\begin{aligned} q_1(h) = & \frac{\rho(h)}{Q} \int_{E_{\min}}^{E_2(h)} D_1(E_0) \times 231 \times \\ & \times \left[200^{1.77} + 231 \times 1.77 \left(\frac{850^{1.53} - 200^{1.53}}{68 \times 1.53} + \frac{E_0 - 850}{1.91} - h \right) \right]^{\frac{0.77}{1.77}} dE_0 + \\ & + \frac{\rho(h)}{Q} \int_{E_2(h)}^{E_1(h)} D_1(E_0) \times 68 \times \left[850^{1.53} + 68 \times 1.53 \times \right. \\ & \times \left. \left(\frac{E_0 - 850}{1.91} - h \right) \right]^{\frac{0.53}{1.53}} dE_0 + \\ & + \frac{\rho(h)}{Q} \int_{E_1(h)}^{5000} D_1(E_0) \times 1.91 dE_0 \end{aligned}$$

Case 4: $E_4(h) \leq E_{\min} < E_3(h) < E_2(h) < E_1(h) < 5000$ MeV

$E_4(h)$ is obtained as follows:

$$E_4(h) = 850 + 1.91 \left(h - \frac{0.15^{0.5} - (kT)^{0.5}}{1.285 \times 10^3} - \frac{200^{1.77} - 0.15^{1.77}}{231 \times 1.77} - \frac{850^{1.53} - 200^{1.53}}{68 \times 1.53} \right)$$

The corresponding electron production rate is summed over four energy intervals:

$$\begin{aligned} q_1(h) = & \frac{\rho(h)}{Q} \int_{E_{\min}}^{E_3(h)} D_1(E_0) \times 2.57 \times 10^3 \times \left[0.15^{0.5} + \right. \\ & + 1.285 \times 10^3 \left(\frac{200^{1.77} - 0.15^{1.77}}{231 \times 1.77} + \frac{850^{1.53} - 200^{1.53}}{68 \times 1.53} + \right. \\ & + \left. \left. \frac{E_0 - 850}{1.91} - h \right) \right]^{2/2} dE_0 + \\ & + \frac{\rho(h)}{Q} \int_{E_3(h)}^{E_2(h)} D_1(E_0) \times 231 \times \left[200^{1.77} + 231 \times 1.77 \left(\frac{850^{1.53} - 200^{1.53}}{68 \times 1.53} + \right. \right. \\ & + \left. \left. \frac{E_0 - 850}{1.91} - h \right) \right]^{\frac{0.77}{1.77}} dE_0 + \\ & + \frac{\rho(h)}{Q} \int_{E_2(h)}^{E_1(h)} D_1(E_0) \times 68 \times \left[850^{1.53} + 68 \times 1.53 \left(\frac{E_0 - 850}{1.91} - h \right) \right]^{\frac{0.53}{1.53}} dE_0 + \end{aligned}$$

$$+ \frac{\rho(h)}{Q} \int_{E_1(h)}^{5000} D_1(E_0) \times 1.91 dE_0$$

Case 5: If $E_{\min} \leq E_4(h) \rightarrow E_{\min} = E_4(h)$. This case is similar to Case 4 for particles with initial energy in the interval (200 - 850 MeV).

Conclusion. More accurate expressions for CR energy decrease $E(h)$ and electron production rate profiles $q(h)$ are derived in the present paper. We introduce more high energy intervals 200 - 850 MeV and 850 - 5000 MeV, which are important for galactic CR [11, 12]. The obtained formulas allow comparatively easy computer programming. The integrand in $q(h)$ gives the possibility for application of adequate numerical methods, for example Romberg method, or Gauss quadrature, for the solution of the mathematical problem [13, 14]. The boundaries of integration do not present singularities. On this way the process of interaction of cosmic ray particles with the upper, middle and lower atmosphere will be described much more realistically. An improvement of the model will include the account of the atmospheric cut-offs $E_A(h, Z)$ [15].

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