Modeling of solar energetic particles in interplanetary space

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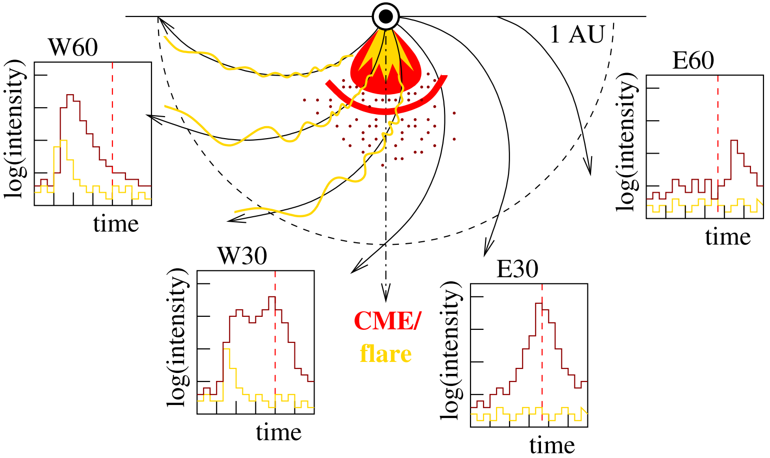
Outline

Solar energetic particle events

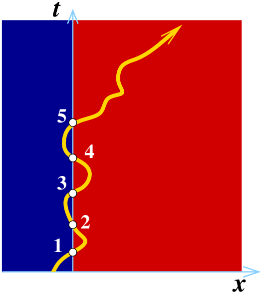
- Solar Energetic Particle (SEP) events related to
  - X-ray flares
  - Coronal Mass Ejections (CMEs)
- SEP event categories
  - impulsive events
    - related to impulsive **flares** ("flare-acceleration")
    - enriched in electrons, $^{3}\mathrm{He}$ and heavy ions
    - duration from hours to days; "low" intensities
  - gradual events
    - related to gradual flares and **CMEs** ("shock-acceleration")
    - typically "normal abundances"
    - duration from days to a week; "high" intensities
  - hybrid events
    - flare- and shock-acceleration observed simultaneously, and/or
    - shock-acceleration of supra-thermals remnant from prior impulsive flares

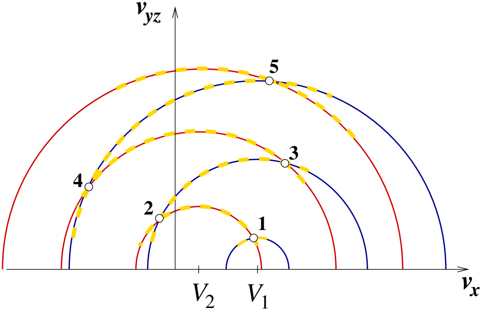
SEP events vs. source longitude



SEP acceleration processes

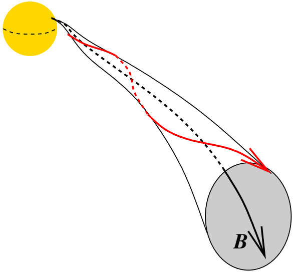
- Coronal sources ($r_\mathrm{s} \sim R_\odot$)
  - flares (reconnection E-fields, stochastic acceleration)
  - coronal shock waves (shock acceleration)
- Interplanetary sources ($r_\mathrm{s} \gg R_\odot$)
  - interplanetary shock waves
- Modeled either physically or phenomenologically

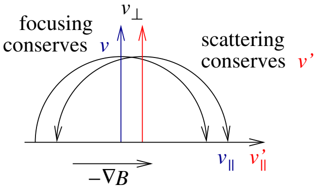




SEP transport processes in IP space

- Gyrating around and streaming along the IMF field lines
- $\boldsymbol{E} \times \boldsymbol{B}$ drift $\Rightarrow$ co-rotation
- Adiabatic focusing (mirroring)
- Scattering off magnetic irregularities $\longrightarrow$
  - diffusion in pitch angle
  - spatial diffusion
- Convection with the scattering centers
- Adiabatic deceleration





Test-particle models

- IP transport models
- IP-shock acceleration models

Diffusion–convection models – Parker's equation

Particle transport under strong scattering (*Parker* 1965)

$$\begin{aligned}\frac{\partial f}{\partial t} = & -\boldsymbol{V}\cdot\nabla f && \text{convection}\\ & +\frac{p}{3}(\nabla\cdot\boldsymbol{V})\frac{\partial f}{\partial p} && \text{adiabatic deceleration}\\ & +\nabla\cdot(\boldsymbol{\kappa}\cdot\nabla f) && \text{spatial diffusion}\end{aligned} \qquad \text{(PE)}$$

f omnidirectional distribution function

$\boldsymbol{V}$ solar wind speed

$\boldsymbol{\kappa}$ spatial diffusion tensor, $\kappa_{ij} = \kappa_{ij}^{(\mathrm{s})} + \kappa_{ij}^{(\mathrm{a})}$

- symmetric part: $\kappa_{ij}^{(\mathrm{s})} = \kappa_\perp\delta_{ij} + (\kappa_\parallel - \kappa_\perp)b_i b_j$, $b_i = B_i/B$.
- antisymmetric part (drifts): $\kappa_{ij}^{(\mathrm{a})} = \frac{1}{3} r_\mathrm{L} v\, \epsilon_{ijk} b_k$.

Notes: (PE) assumes

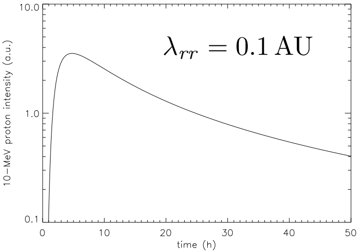
- quasi-isotropic particle distributions, $\lambda_\parallel \equiv 3\kappa_\parallel/v \ll r$
- frozen-in fluctuating fields
  - convection velocity neglects wave transport
  - stochastic acceleration neglected

Diffusive SEP transport

(PE) can be further simplified for $>$ 10-MeV solar protons:

- drifts, adiabatic deceleration and convection neglected
- if $\kappa_\perp = 0$ **assumed**
  - $\Rightarrow$ diffusion along field lines:

$$\boldsymbol{\kappa} = \kappa_\| \boldsymbol{b}\boldsymbol{b}$$



$\Rightarrow$ radial diffusion model, $\kappa_{rr} = \kappa_\| \cos^2\psi = \frac{1}{3}\lambda_{rr} v$

$$\therefore \quad \frac{\partial n_p}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \kappa_{rr} \frac{\partial n_p}{\partial r}\right); \qquad n_p \equiv \frac{d^4 N}{d^3 r\, dp} = 4\pi p^2 f = 4\pi I$$

$\Rightarrow$ **Green's function** (e.g. *Wibberenz et al.* 1989)

$$n_p(r,t) = \frac{n_0}{r^3}\left(\frac{r^2}{a^2 \kappa_{rr} t}\right)^{\frac{3}{a}} \exp\left\{-\frac{r^2}{a^2 \kappa_{rr} t}\right\}, \quad n_0(p) = \frac{a}{\Gamma(\frac{3}{a})}\left.\frac{dN}{dp}\right|_{\mathrm{s}}$$

for $\boxed{\lambda_{rr} \propto r^b}$ with $a = 2 - b$ and $b < 2$.

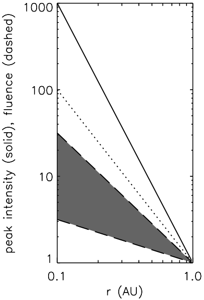
Scaling laws from diffusive transport

Typically, best fit for SEP data given by $b \in [-0.5, 0.5] \Rightarrow$ Green's function

$$n_p(r,t) = \frac{n_0(p)}{r^3}\left(\frac{r^2}{(2-b)^2\kappa_{rr}t}\right)^{\frac{3}{2-b}} \exp\left\{-\frac{r^2}{(2-b)^2\kappa_{rr}t}\right\}.$$

$\Rightarrow$ **Scaling laws** for short SEP injections from the Sun (impulsive events and gradual events with $\phi \gtrsim 60°$W):

$$n_s(p) \propto I(r_\otimes, E, t_{max})$$
$$I(r, t_{max}) \propto r^{-3}$$
$$\mathcal{F}_p(r) = \int_0^\infty I\,dt \propto \frac{n_s(p)}{r\kappa_{rr}(r,p)} \propto r^{-b-1}$$



$\Rightarrow$ r^{-2}-scaling **not applicable** to intensity or fluence

$\Rightarrow$ fluence spectrum not proportional to source spectrum

Focused transport equation

Weak scattering ($\lambda_{rr} \gtrsim 0.2\, r$) $\Rightarrow$ anisotropic f $\Rightarrow$ (*Roelof* 1969)

$$\frac{\partial f}{\partial t} = -v\mu \frac{\partial f}{\partial s} \qquad \text{streaming}$$
$$-(1-\mu^2)\frac{v}{2L}\frac{\partial f}{\partial \mu} \qquad \text{focusing} \qquad \text{(FT)}$$
$$+\frac{\partial}{\partial \mu}\left(D_{\mu\mu}\frac{\partial f}{\partial \mu}\right) \qquad \text{pitch-angle diffusion}$$

f (gyro-tropic) distribution function

s particle position along the field line (co-rotating frame)

v, μ particle speed and pitch-angle cosine (co-rotating frame)

L focusing length, $L^{-1} = -\frac{1}{B}\frac{\partial B}{\partial s}$.

$D_{\mu\mu}$ pitch-angle diffusion coefficient

Note: v only a parameter so the phase space is 2D: (z, μ)

Focused transport equation – solar wind effects

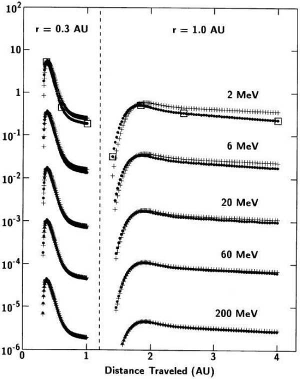
If $V \neq 0$ is taken into account (*Ruffolo* 1995)

$$\begin{aligned}
\frac{\partial f}{\partial t} &= -(v\mu + V)\frac{\partial f}{\partial s} && \text{streaming \& convect.} \\
&\quad - \left(\frac{v + \mu V}{2L}\right)(1-\mu^2)\frac{\partial f}{\partial \mu} && \text{focusing} \\
&\quad + \left(\frac{1-\mu^2}{2L}V + \frac{dV}{ds}\mu^2\right)p\frac{\partial f}{\partial p} && \text{adiab. deceleration} \\
&\quad + \frac{\partial}{\partial \mu}\left(D_{\mu\mu}\frac{\partial f}{\partial \mu}\right) && \text{pitch-angle diffusion}
\end{aligned} \qquad \text{(FT)}$$

- f (gyro-tropic) distribution function
- s particle position along the field line (co-rotating frame)
- p, v, μ particle momentum, speed and pitch-angle cosine (wind frame)
- L focusing length, $L^{-1} = -\frac{1}{B}\frac{\partial B}{\partial s}$.
- V solar wind speed (co-rotating frame)
- $D_{\mu\mu}$ pitch-angle diffusion coefficient

Which focused transport equation should we use?

- *Ruffolo* (1995) and *Kocharov et al.* (1998) investigated the solar-wind effects in (FT)
- Effects pronounced at low ($E \sim 2$ MeV) energies, where
  - C speeds up transport early in the event
  - AD and C harden the spectrum late in the event
  - Effects less pronounced closer to the Sun



Ruffolo (1995)

- At $E \gtrsim 10$ MeV, we may neglect C and AD effects rather safely for SpW purposes.

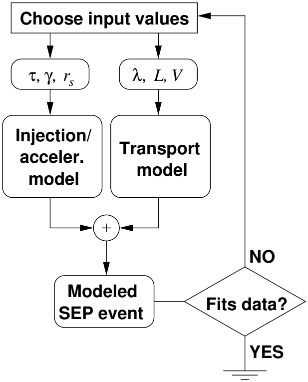
Numerical methods to solve FTE

- Monte-Carlo simulations (e.g., *Kocharov et al.*, *Vainio et al.*)
  - trace individual particles in heliospheric fields
  - scatter (via random generator) elastically in the frame of the scattering center (wind frame)
  - advantages and disadvantages

    ↑ easily implemented and tested
    ↓ rather slowly running codes
- Finite difference schemes (e.g., *Ng et al.*, *Ruffolo*, *Heras et al.*, *Hatzky et al.*)
  - solve FTE directly on a grid
  - advantages and disadvantages

    ↓ more cumbersome numerical methods needed
    ↑ faster codes (than MC)

Elements of SEP event modeling



SEP observations of

- omnidirectional intensity

$$I(E, \boldsymbol{r}, t) \equiv \frac{p^2}{4\pi} \int f(\boldsymbol{p}, \boldsymbol{r}, t)\, d\Omega$$

- anisotropy

$$\xi(E, \boldsymbol{r}, t) \equiv \frac{3 \int \mu f(\boldsymbol{p}, \boldsymbol{r}, t)\, d\Omega}{\int f(\boldsymbol{p}, \boldsymbol{r}, t)\, d\Omega}$$

compared with modeled SEP event.

⇒ Understanding of SEP sources and transport conditions.

Modeling SEP sources

- Focused transport equation + SEP injection/acceleration
- Possible approaches:
  - include source term (*phenomenological*),

$$\frac{\partial f}{\partial t} + v\mu\frac{\partial f}{\partial s} + (1-\mu^2)\frac{v}{2L}\frac{\partial f}{\partial \mu} = \frac{\partial}{\partial \mu}\left(D_{\mu\mu}\frac{\partial f}{\partial \mu}\right) + \boxed{Q(s,p,\mu,t)}$$

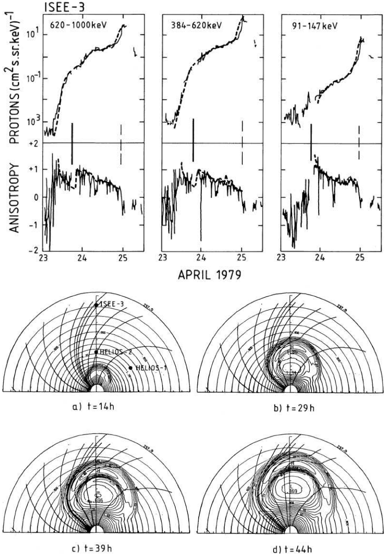
  - use (inner) boundary condition (*phenomenological* or *physical*),

$$f(s_0(t),p,\mu,t)_{\mu>0} = S_0(p,\mu,t) + F\{f(t)_{\mu<0}\}$$

  - include a shock in the solar wind speed (*physical*)

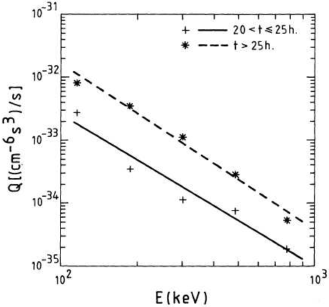
$$\frac{\partial f}{\partial t} = -(v\mu+V)\frac{\partial f}{\partial s} - \left(\frac{v+\mu V}{2L}\right)(1-\mu^2)\frac{\partial f}{\partial \mu} + \frac{\partial}{\partial \mu}\left(D_{\mu\mu}\frac{\partial f}{\partial \mu}\right) + \left(\frac{1-\mu^2}{2L}V + \boxed{\frac{dV}{ds}\mu^2}\right)p\frac{\partial f}{\partial p}$$

Phenomenological source modeling



Example: *Heras et al.* (1992)

- Sources: corona & IP shock
- Injection profiles: $R + Q$
- IP transport: $\lambda_{\|}$ and q



Phenomenological source modeling – key results

- CME-driven shocks are efficient particle accelerators up to hundreds of MeVs (e.g., *Torsti et al.* 1996, *Anttila et al.* 1998)
  - Spectra $\propto E^{-\gamma} e^{-E/E_0}$ ↑↑ shock acceleration
- The nose region of the shock most efficient in accelerating particles (e.g., *Anttila et al.* 1998)
- Injection rate Q at the shock increases with shock velocity ratio (e.g., *Lario et al.* 1997)
- Magnetic connection of the S/C to the shock essential (e.g., *Heras et al.* 1992)
- Typical values of the interplanetary scattering m.f.p.

$$\lambda_{\parallel} \sim 0.1 - 1 \text{ AU}, \quad \lambda_{rr} = \lambda_{\parallel} \cos^2 \psi \propto r^b, \; b \sim 0$$

- Useful approach for creating synthetic SEP events (Space Weather)
  - Engineering model SOLPENCO (*Aran et al.* 2004)

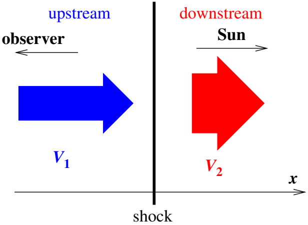
Physical approach – particle acceleration at shocks

- Diffusive shock acceleration (steady state, shock frame):

$$V\frac{\partial f}{\partial x} - \frac{p}{3}\frac{\partial V}{\partial x}\frac{\partial f}{\partial p} = \frac{\partial}{\partial x}\left(\kappa\frac{\partial f}{\partial x}\right) + Q_0\delta(x)\delta(p-p_0) \quad \text{(1D-PE)}$$

$$\Rightarrow f = \begin{cases} f_2 \exp\{\int_0^x V_1\, dx'/\kappa(x')\}, & x<0 \\ f_2 & x>0 \end{cases}$$

$$f_2(p) = \frac{Q_0\sigma}{V_1}\left(\frac{p}{p_0}\right)^{-\sigma}, \quad \sigma = \frac{3V_1}{V_1-V_2}$$



$\Rightarrow$ Transport equations contain the necessary terms to model IP particle acceleration!

- Why don't we use them?

Diffusive shock acceleration – role of turbulence

- DSA

$$f = \begin{cases} f_2 e^{\int_0^x V_1\,dx'/\kappa(x')}, & x < 0 \\ f_2 & x > 0 \end{cases}$$

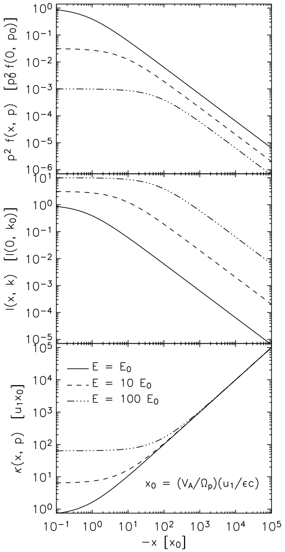
$$f_2(p) = \frac{Q_0\sigma}{V_1}\left(\frac{p}{p_0}\right)^{-\sigma}, \; \sigma = \frac{3\,V_1}{V_1 - V_2}$$

- Key parameter: particle acceleration rate

$$\frac{\dot p}{p} \sim \frac{V_{\rm sh}^2}{\kappa} \sim \frac{V_{\rm sh}^2}{\lambda v} > \frac{V_{\rm sh}}{r}$$

$$\Rightarrow \lambda < r\frac{V_{\rm sh}}{v} \qquad \uparrow\downarrow \lambda_{\rm obs}$$

$\Rightarrow$ Self-generated waves (Bell 1978)



Self-consistent models

- Generation of waves by SEPs
- Self-consistent models of SEP acceleration at IP Shocks

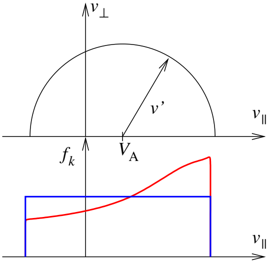
Wave–particle interactions

- Particles scattered by MHD (Alfvén) waves with

$$\omega = \pm V_A k$$

- Cyclotron resonance (and a useful approximation)

$$k = \frac{\omega_c}{v\mu \pm V_A} \simeq \frac{\omega_c}{v'}$$



- Particle scattering leads to isotropy in wave frame
  - particle energy density in plasma frame increases/decreases
  - $\Rightarrow$ wave intensity $I(k)$ decreases/increases
- Scattering rate and Wave-growth rate

$$D_{\mu\mu} \simeq \frac{\pi}{2}\omega_{ci}(1-\mu^2)\frac{[k\,I(k)]_{k=m\omega_c/p}}{B^2};\ \lambda = \frac{3v}{8}\int_{-1}^{+1}\frac{(1-\mu^2)^2}{D_{\mu\mu}}d\mu$$

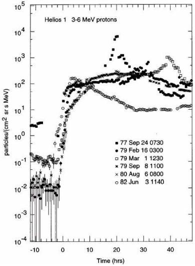
$$\gamma(k) \simeq \frac{\pi}{2}\omega_{ci}\frac{[p^3 S(p)]_{p=m\omega_c/k}}{V_A n_e};\ S = \int v\mu f\, d\Omega$$

Streaming-limited intensities

- Observationally (*Reames & Ng* 1998)

$$I_p \lesssim \frac{5 \cdot 10^2}{\text{cm}^2\ \text{sr s MeV}}$$

at MeV energies before shock arrival



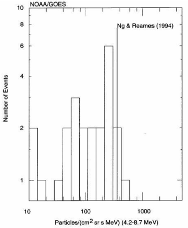
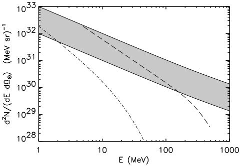
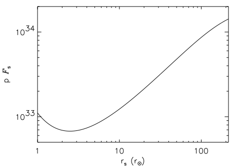
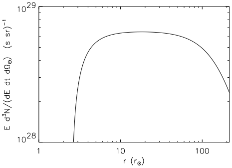


FIG. 2.—A histogram of streaming-limited plateau intensities of 4.2–8.7 MeV protons from the *GOES* spacecraft is compared with the calculated (Ng & Reames 1994) maximum plateau intensity at 1.0 AU.

- Can be understood by coronal trapping in self-generated turbulence (*Ng & Reames* 1994; *Vainio* 2003)
- Model predicts the time-evolution of abundance ratios of high-to-low rigidity ions as well (*Ng et al.* 1999)







Ion acceleration at IP shocks

- Problem with Bell's (1D) theory: no particle escape upstream
- Possible cures:
  - focusing
  - time dependence
- Rice et al., Zank et al., Li et al.
  - Bell's analytical model in a small region around the shock
  - ad-hoc free escape boundary upstream
  - diffusive transport downstream and focused transport upstream
  - Quasi-steady state model
- Lee (2005)
  - Uses $f_- = \int_{-1}^{0} f\, d\mu$ and $f_+ = \int_{0}^{+1} f\, d\mu$ instead of $\int_{-1}^{+1} f\, d\mu$

  $\Rightarrow$ self-consistent wave-generation and escape by focusing
  - Quasi-steady state model
- Vainio & Laitinen (in preparation)
  - fully time dependent MC simulations (xpdf/acroread)

Conclusions

- SEP transport in the interplanetary space in principle well understood
  - test-particle modeling
  - self-generated waves
- Numerical methods to solve the transport equations
  - Finite difference solutions
  - Monte Carlo simulations
- Ad-hoc source modeling has yielded empirical information of
  - interplanetary transport conditions
  - interplanetary shock acceleration
- Physical modeling of interplanetary acceleration has experienced several breakthroughs over the last few years
  - role of focusing in particle escape
  - importance of full time dependence in self-consistent modeling

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